

## EQUAL RATE OF RETURNS UNDER THE OBJECTIVE DEMAND FUNCTION\*

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### I Introduction

This paper is concerned with the existence of the equal rate of returns within same industry under the objective demand function. Nikaido<sup><4></sup> is the first, to my best knowledge, who constructed the objective demand function to analyse the mutual interdependence of the economic agents in non-cooperative, monopolistic competition.

In the traditional analysis for monopolistic competition, monopolists are assumed to have their own subjective demand functions in their minds and make their optimal plannings based on the functions. Whatever demand functions in subjective base they might have and make their plannings, their intentions would not be realized if the planned outputs are not sold out with the incomes that economic agents earn by participating in the plannings. The objective demand function forming a striking contrast to the perceived or subjective demand functions is the one on which the demand have consistent income backgrounds, in other words, each demand for goods on the curve is effective one coming from the incomes of the agents in the national economy<sup>1)</sup> (see equation (12)).

We carry out our analysis in the leontief type economy as done in Nikaido's but our economy differs from his in these respects that we have a finite number of potential firms within every industry (see A. I) and that those potential firms are, furthermore, assumed to be able to survive in the price competition (see A. IV).

We show within this framework with some other assumptions the existence of the relative shares which yield the equal rate of returns rather than the equal expected profits among those potential firms belonging to same industry.

### II Assumptions & Notations

#### A. I.

$T_j \equiv \{1, \dots, t_j\}$  : a set of finite number of potential firms in industry  $j$ .

Let

$$a_{ij}(\tau_j), l_j(\tau_j) \quad (\tau_j \in T_j, i, j=1, \dots, n)$$

be input coefficient and labor input of firm  $\tau_j (\tau_j \in T_j)$ , respectively.  $a_{ij}(\tau_j)$  is the amount of good necessary for a unit production of good  $j$ , which is assumed nonnegative and constant,

$$a_{ij}(\tau_j) \geq 0 \quad (\tau_j \in T_j, i, j=1, \dots, n).$$

Labor input is always indispensable, so that

$$l_j(\tau_j) > 0 \quad (\tau_j \in T_j, j=1, \dots, n).$$

Let, further,  $\pi_j(\tau_j) \geq 0 \quad (\tau_j \in T_j, j=1, \dots, n)$

be the expected profit per unit output of the firm  $\tau_j$ .

There is no joint production; so that any firm  $\tau_j (\tau_j \in T_j)$  produces only one good  $j (j=1, \dots, n)$ , employing the current material inputs  $a_{ij}(\tau_j) (i=1, \dots, n)$  and labor input  $l_j(\tau_j)$ .

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The remaining inelegancies are of course mine.

1) A prominent work based on the subjective demand functions is Negishi's<sup><3></sup>.

One of the defects, however, in adopting such demand functions for the analysis of monopolistic competition is to assume that ever monopolistic firm perceives the actual correct demand, though not necessarily the true demand function (see pp. 157-8. Arrow-Hahn<sup><1></sup>).

**A. II.**

The number of potential firms, in general, varies over industries; we assume, however, for simplicity, they are the same for all industries. Let us suppose there are  $m$  such firms in every industry. If  $S$  denotes a set of firms the  $j$ -th entry of which is an element of the set  $T_j$ , then we have  $m$  such finite disjoint set—let it be  $(S_1, \dots, S_m)$ ; in other words each firm of every industry must be an element of one and only one such set.<sup>2)</sup>

**A. III.**

$A_k$  is a  $n \times n$  input coefficients matrix whose  $i$ -th column is the input coefficients of the firm which is the  $i$ -th element of set  $S_k$ . The same applies to the labor input vector  $l_k (\in R_+^n)$  and the expected profit vector  $\pi_k (\in R_+^n)$ .<sup>3)</sup> Each  $A_k (k=1, \dots, m)$  is productive and furthermore indecomposable when necessary.<sup>4)</sup>

**A. IV.**

All the potential firms are equally competitive; when some firm in an industry tries to monopolize the supply of the good by pushing down its price of the good—this is done through setting lower expected profit than before—, other firms in the industry can remain in the markets and continue their productions without incurring any loss. Suppose, for example, all the firms in the set  $S_1$  try to drive out all the other firms by keeping their expected profit  $\pi_1=0$ , other firms then in this situation can operate their production with zero profit. Let  $\sigma$  be the price when  $\pi_1=0$ , the assumption says that

$$\sigma = \sigma A_1 + l_1 \tag{1)^5}$$

implies

$$\sigma = \sigma A_i + l_i \quad (i=2, \dots, m).$$

**A. V.**

Both laborers and capitalists spend all their incomes on purchase of the commodities. Let us denote the intended demands for commodities of laborers and capitalists by  $F(p, 1)$  ( $\in R_+^n$ ) and  $G(p; I_1, \dots, I_n)$  ( $\in R_+^n$ ) respectively; when  $I_i$  is the expected incomes of the capitalists of industry  $i$ .

Capitalists receive their incomes from the profits they gain in their productions; they will be, as a whole,

2) If  $\tau_j \in T_j$ , there is some  $S_k$  such that

$$\begin{matrix} (* \dots * \tau_j * \dots *) \in S_k \\ \uparrow \\ \text{the } j\text{-th element} \end{matrix}$$

and

$$(* \dots * \tau_j * \dots *) \notin S_j \text{ for } \forall S_j \neq S_k.$$

3) Suppose

$S_k \equiv (\tau_1, \tau_2, \dots, \tau_j, \dots, \tau_n)$  ( $\tau_j \in T_j, j=1, \dots, n$ ).  $A_k, l_k$  and  $\pi_k$  are then given by

$$A_k \equiv \begin{pmatrix} a_{n1}(\tau_1) & \dots & a_{n1}(\tau_n) \\ \vdots & & \vdots \\ a_{n1}(\tau_1) & a_{ij}(\tau_j) & a_{nn}(\tau_n) \end{pmatrix}$$

$$l_k \equiv (l_1(\tau_1) \dots l_n(\tau_n))$$

$$\pi_k \equiv (\pi_1(\tau_1) \dots \pi_n(\tau_n)).$$

4) A  $n \times n$  Matrix  $A (n \geq 2)$  is said productive, when there is a non-negative vector  $\bar{x} (\geq 0)$  such that  $A\bar{x} < \bar{x}$ .

A  $n \times n$  matrix  $A$  is said to be indecomposable if no permutation matrix  $\Pi$  does

$$\Pi A \Pi^{-1} = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \text{ where } A_{11}, A_{22} \text{ are square.}$$

See Debreu & Herstein <2> for the properties of such matrices.

5) The prices are normalised so that  $W$ , the price of labor service, is equal to unity.

$$\pi x^{6)}$$

where  $x \equiv (x_i) \in R_+^n$  is the output vector,  $x_i$  representing the output level of good  $i$ . On the other hand, laborers' incomes come for their supplies of labor service, letting  $L(p, 1) (> 0)$  the intended supply of the service as a whole given the price  $p$ , their aggregate income is  $F(p, 1)$ , wage rate  $w$  being set to unity. Therefore we have, by virtue of the assumption,

$$\begin{aligned} L(p, 1) &= p \cdot F(p, 1) \\ \pi \cdot x &= p \cdot G(p; I_1, \dots, I_n). \end{aligned} \quad (2)$$

#### A. VI.

All the relevant functions  $L(p, 1)$ ,  $F(p, 1)$  and  $G(p; I_1, \dots, I_n)$  are continuous in their arguments.

#### A. VII.

$$\Gamma_k \equiv \begin{pmatrix} \theta_1(\tau_1) & & 0 \\ & \ddots & \\ 0 & & \theta_n(\tau_n) \end{pmatrix} \quad (k=1, \dots, m)$$

is a  $n \times n$  diagonal matrix, where

$$\theta_j(\tau_j) \geq 0, \quad \sum_{\tau_j \in T_j} \theta_j(\tau_j) = 1 \quad (j=1, \dots, n) \quad (3)$$

$\theta_j(\tau_j)$ , representing the relative share of the firm  $\tau_j$  in industry  $j$ .

If the set of firms  $S_k$  consists of such firms  $(\tau_1, \dots, \tau_n)$ , the matrix  $\Gamma_k$  shows the relative shares of the firms in the set  $S_k$ . In matrix form, (3) becomes,

$$\Gamma_k \geq 0 \quad (k=1, \dots, m) \quad (3)'$$

$$\sum_{k=1}^m \Gamma_k = E \quad (E: \text{identity matrix of order } n).$$

When the total production  $x (\in R_+^n)$  in the economy is given,  $\Gamma_k x (\in R_+^n)$  is the output of the firms in the set  $S_k$ .

### III Model Building

Equilibrium price  $p$  must satisfy

$$p = pA_k + l_k + \pi_k \quad (4)^7)$$

given the expected profits  $\pi_k (> 0)$  of the firms in  $S_k (k=1, \dots, m)$ .

Postmultiplying equ. (4) by  $\Gamma_k$  and summing over  $k$ , we obtain, by virtue of (3)'

$$p = pA(\theta) + l(\theta) + \pi(\theta) \quad (5)$$

where  $A(\theta) \equiv \sum_{k=1}^m A_k \cdot \Gamma_k$

$$l(\theta) \equiv \sum_{k=1}^m l_k \cdot \Gamma_k$$

$$\pi(\theta) \equiv \sum_{k=1}^m \pi_k \cdot \Gamma_k.$$

Similarly, equilibrium output vector  $x (\in R_+^n)$  satisfies in output side

6)  $\pi \cdot x$  is the product of two vectors  $\pi$  and  $x$ . The vectors are well defined so that the calculation of product is possible. For example  $\pi$  is taken as a row vector and  $x$  a column vector. The same applies to the relevant calculations below.

7) We have a positive price  $p > 0$ , given  $\pi_k > 0$ , because  $A_k$  is productive by virtue of (A. IV); in fact  $p$  is given by

$$p = l_k [I - A_k]^{-1} + \pi_k [I - A_k]^{-1} = \sigma + \pi_k [I - A_k]^{-1} > 0.$$

$$x = A(\theta)x + F(p, 1) + G(p; I_1, \dots, I_n) \quad (6)$$

where  $I_i = \pi_i(\theta) \cdot x_i$  ( $\pi_i(\theta)$  and  $x_i$  are the  $i$ -th elements of the vectors  $\pi(\theta)$  and  $x$  respectively.)

The output vector  $x$  is bounded by the available labor service.

$$l(\theta) \cdot x = L(p, 1). \quad (7)^{8)}$$

Now we will show under the pair  $(p, \theta)$  the existence of output vector  $X$  which satisfies equ. (6) under equ. (7) and budget equations.

$$p \cdot F(p, 1) = L(p, 1)$$

$$p \cdot G(p, I_1, \dots, I_n) \equiv \sum_{i=1}^n I_i = \pi(\theta) \cdot x. \quad (2)'$$

Let

$$T(p, \theta) \equiv \{x \mid x \in R_+^n, l(\theta) \cdot x \equiv L(p, 1)\}.$$

According to the definition  $l(\theta)$  and (A. I),  $l(\theta) (\in R_+^n) > 0$  and  $L(p, 1) > 0$  by (A. V); hence  $T(p, \theta)$  is a compact set.

Define

$$f(p, \theta; x) \equiv A(\theta) \cdot x + F(p, 1) + G(p, \pi_1(\theta)x_1, \pi_2(\theta)x_2, \dots, \pi_n(\theta)x_n)$$

8) Define  $C(p, \theta)$  as

$$C(p, \theta) \equiv \{c \mid c \geq 0, x = A(\theta)x + a(p, 1) l(\theta) \cdot x + c, l(\theta) \cdot x \leq L(p, 1)\}$$

where  $a(p, 1) (\in R_+^n)$  is the commodity (column) vector necessary for laborer to maintain his unit labor supply and  $C$  is the commodity vector available for capitalists.

We assume, as proposed in (A. V),

$$p \cdot a(p, 1) = 1$$

$$\pi(\theta) \cdot x = p \cdot c. \quad (n.1)$$

As  $A(\theta)$  is non-negative and indecomposable, according to its construction and (A. III) so is all the more,  $A(\theta) + a(p, 1) l(\theta)$ . Therefore we have a positive scalar  $\lambda (> 0)$  which is the largest in modulus among the eigen roots of the matrix and a positive vector  $y (> 0)$  such that

$$[A(\theta) + a(p, 1) l(\theta)]y = \lambda y. \quad (n.2)$$

Premultiplying this equ. by the vector  $p$  given by equ. (4) or (5),

$$\begin{aligned} \lambda p y &= p [A(\theta) + a(p, 1) l(\theta)] y \\ &= [p A(\theta) + l(\theta)] y \quad (\because p a(p, 1) = 1) \\ &= [p - \pi(\theta)] y \quad (\text{cf. equ. (5)}). \end{aligned}$$

Therefore,

$$(1 - \lambda) p y = \pi(\theta) y.$$

As  $\pi(\theta) > 0$ ,  $p > 0$  and  $y > 0$ , we have

$$\lambda > 1.$$

Hence, there exists the inverse matrix of  $[I - A(\theta) - a(p, 1) l(\theta)]$  and further it is a positive matrix;  $[I - A(\theta) - a(p, 1) l(\theta)]^{-1} > 0$ .

The output vector  $x = x(p, \theta; c)$  is given by

$$x(p, \theta; c) = [I - A(\theta) - a(p, 1) l(\theta)]^{-1} c \quad \text{for } \forall c \in C(p; \theta).$$

Suppose

$$l(\theta) x(p, \theta; c^0) = l(\theta) [I - A(\theta) - a(p, 1) l(\theta)]^{-1} c^0 < L(p, 1)$$

for some  $c^0 \in C(p, \theta)$ , then we have a scalar  $t (> 1)$ , for which

$$l(\theta) x(p, \theta; t c^0) = L(p, 1) \quad t c^0 \in C(p, \theta). \quad (n.3)$$

In fact if we take

$$t \equiv \frac{L(p, 1)}{l(\theta) [I - A(\theta) - a(p, 1) l(\theta)]^{-1} c^0}$$

equ. (n.3) holds.

Therefore from now on capitalists are supposed to expand their output so far as all the available labor service gets absorbed in production, because they can more more available commodities for their own.

The commodities distributed among laborers, under the supposition, are

$$\begin{aligned} a(p, 1) l(\theta) x(p, \theta; c) &= a(p, 1) L(p, 1) \\ &= F(p, 1). \end{aligned}$$

These justify our consideration in the system of equs. (6) and (7).

$$g(p, \theta; x) \equiv \frac{L(p, 1)}{l(\theta)f(p, \theta; x)} \cdot f(p, \theta; x). \quad (8)^9$$

Because  $T(p; \theta)$  is compact for the given pair  $(p, \theta)$ ,  $g(p, \theta; x)$  is continuous in  $x$  by virtue of (A. VI), and  $g(p, \theta; x)T \in (p, \theta)$  for any  $x \in T(p, \theta)$ , we have applying the Brouwer's fixed point theorem, a point  $x^* (\in T(p, \theta))$  such that

$$\begin{aligned} x^* &= g(p, \theta; x^*) \\ &\equiv \frac{L(p, 1)}{l(\theta)f(p, \theta; x^*)} f(p, \theta; x^*). \end{aligned} \quad (9)$$

Premultiplying this by the price  $p$  given by equ. (4)

$$px^* = \frac{L(p, 1)}{l(\theta)f(p, \theta; x^*)} pf(p, \theta; x^*). \quad (10)$$

Postmultiplying equ. (5) by  $x^*$ , we have

$$\begin{aligned} px^* &= pA(\theta)x^* + l(\theta)x^* + \Pi(\theta)x^* \\ &= pA(\theta)x^* + L(p, 1) + \pi(\theta)x^* \quad (\text{cf. equ. (7)}) \\ &= pA(\theta)x^* + p \cdot F(p, 1) + pG(p, \pi_1(\theta)x_1^*, \dots, \pi_n(\theta)x_n^*) \quad (\text{cf. equ. (2)'}) \\ &= p[A(\theta)x^* + F(p, 1) + G(p, \pi_1(\theta)x_1^*, \dots, \pi_n(\theta)x_n^*)] \\ &= pf(p, \theta; x^*). \end{aligned} \quad (11)$$

We obtain, by equs. (10) and (11), as well as  $px^* > 0$

$$L(p, 1) = l(\theta) \cdot f(p, \theta; x^*).$$

This implies, by virtue of equ. (9)

$$x^* = f(p, \theta; x^*)$$

$$= A(\theta)x^* + F(p, 1) + G(p, \pi_1(\theta)x_1^*, \dots, \pi_n(\theta)x_n^*)$$

with

$$l(\theta)x^* = L(p, 1). \quad (12)$$

We have proved the existence of the equilibrium output  $x^* = x(p, \theta)$  for the given pair  $(p, \theta)$ .

It is easy to see that we can always construct such  $x^*$  for any price  $p (> 0)$  with equ.(5) behind once the relative share  $\theta$  is given. The relation of the equilibrium output and the prices is nothing but the objective demand function. The output  $x^* (\in R_+^n)$  always matches its demand: part of it is demanded as intermediate goods among firms — the first term of equ. (12) corresponds to this demand —, another part  $F(p, 1)$  is consumed by the laborers as a whole; and the others remain for the consumption of the capitalists whose incomes come from the profits they get engaged in the productions. When the capitalists attempt to produce this amount of goods it just meets its demand; the source of which just stems from the agents' engagements, there is a consistent flow of goods; no obstacle is there. Now for the uniqueness of the output vector  $x = x(p, \theta)$ , we add a further assumption to the previous ones.

#### A. VIII.

The demand function  $G(p, \pi_1(\theta)x_1, \dots, \pi_n(\theta)x_n)$  of the capitalists is of the form,

$$\begin{aligned} G(p, \pi_1(\theta)x_1, \dots, \pi_n(\theta)x_n) &= g(p) \cdot \sum_{i=1}^n \pi_i(\theta)x_i(p, \theta) \\ &= g(p)\pi(\theta)x(p, \theta). \end{aligned} \quad (13)^{10}$$

By virtue of (A. V), the spending pattern of the capitalists will be specified as

$$p \cdot g(p) = 1. \quad (14)$$

It is easy to show, under the equ. (14), by the same method as in note 8, that there is the inverse matrix of  $[I - A(\theta) - g(p)\pi(\theta)]$  and further it is positive

9) That  $p > 0$  and  $L(p, 1) > 0$  together with  $F(p, 1) \in R_+^n$  implies, by virtue of (2)'  $F(p, 1) \geq 0$ , which, in turn implies  $l(\theta)F(p, 1) > 0$ , because  $l(\theta) > 0$ . Therefore  $g(p, \theta; x)$  is well-defined for any  $x \in T(p, \theta)$ .

10) If all the capitalists' utility functions are same and homothetic, we have such a demand function.

$$[I - A(\theta) - g(p)\pi(\theta)]^{-1} > 0. \quad (15)$$

We have, by equs. (12), (13), and (15),

$$x(p, \theta) = [I - A(\theta) - g(p)\pi(\theta)]^{-1} F(p, 1). \quad (16)$$

The above equ. (16) shows that equilibrium output vector  $x = x(p, \theta)$  is not only unique but also continuous in  $\theta$ , because  $[I - A(\theta) - g(p)\pi(\theta)]^{-1}$  is unique and continuous in  $\theta$ .

So far, no mention has been made of the existing capital stocks in the economy.

It is a plausible assumption some amounts of capital stocks are installed in the sectors, underlying the input coefficients and labor inputs.

Suppose the firms in a set,  $S_1$ , are more abundant in capital than those in some other set,  $S_2$ . If the input coefficients are decreasing functions of the capital stocks, the supposition results in

$$A_1 < A_2. \quad (17)$$

By virtue of <A. IV>, equ. (7) becomes

$$\begin{aligned} p &= l_i [I - A_i]^{-1} + \pi_i [I - A_i]^{-1} \\ &= \sigma \quad + \pi_i [I - A_i]^{-1} \quad (i=1, 2). \end{aligned}$$

So, we have

$$\pi_1 [I - A_1]^{-1} = \pi_2 [I - A_2]^{-1}.$$

The difference of the expected profits between the firms in the distinct sets of the firms is given by

$$\begin{aligned} \pi_1 - \pi_2 &= \pi_1 - \pi_1 [I - A_1]^{-1} [I - A_2] \\ &= \pi_1 [I - A_1]^{-1} [(I - A_1) - (I - A_2)] \\ &= \pi_1 [I - A_1]^{-1} (A_2 - A_1) \end{aligned} \quad (18)$$

which is positive under the indecomposability of  $A_1$ ,  $\pi_1 > 0$  and in equ. (17).

Similarly we can show under the in equ. (17),

$$l_1 > l_2.$$

The mentioned above means that if the firms in the set  $S_1$  keep more capital stocks than those in the set  $S_2$ , the firms in  $S_1$  are higher than those in  $S_2$  in expected drofits and the former employ more labor service than the latter when the price  $p$ , given by equ. (4), prevails in the whole economy. Therefore there are various expected profits among firms even within the same industry depending upon the capital stocks. There is, however, a set of relative shares which yield a equal rate of returns within same industry.

Let  $\gamma_j(\tau_j)$  be the rate of returns of firm  $\tau_j (\tau_j \in T_j)$ . It is, by definition,

$$\begin{aligned} \gamma_j(\tau_j) &\equiv \frac{\pi_j(\tau_j)\theta_j(\tau_j)x_j(p, \theta)}{\theta_j(\tau_j)x_j(p, \theta)[\sum_i a_{ij}(\tau_j)p_i + l_j(\tau_j)] + Q_j(\tau_j)} \\ &= \frac{\theta_j(\tau_j)}{\frac{\theta_j(\tau_j)}{\pi_j(\tau_j)[\sum_i a_{ij}(\tau_j)p_i + l_j(\tau_j)]} + \frac{Q_j(\tau_j)}{\pi_j(\tau_j)x_j(p, \theta)}} \end{aligned} \quad (19)^{11)}$$

Define the simplices

$M_j (j=1, \dots, n)$  and  $M$ :

$$M_j \equiv \{\theta_j(\tau_j) \mid \theta_j(\tau_j) \geq 0, \sum_{\tau_j \in T_j} \theta_j(\tau_j) = 1\} \quad (j=1, \dots, n)$$

$$M \equiv \prod_{j=1}^n M_j. \quad (20)^{12)}$$

11)  $Q_j(\tau_j)$  is the value of the capital stock of firm  $\tau_j (\tau_j \in T_j)$  in terms of wage bill, and assumed positive.

$\gamma_j(\tau_j)$  is well defined, because  $Q_j(\tau_j)$ . As  $[I - A(\theta) - g(p)\pi(\theta)]^{-1} > 0$  (see equ.(15)) and  $F(p, 1) \geq 0$  (see note(9)),  $x(p, \theta) > 0$  by equ.(16).  $\pi_j(\tau_j) > 0$ , by our assumption.

So we can operate the division.

12)  $\prod_{j=1}^n M_j$  is the Cartesian product of the sets  $M_1, \dots, M_n$ .

Let

$$W_j^{\tau_j}(\mathbf{p}, \theta) \equiv \frac{\theta_j(\tau_j)}{\pi_j(\tau_j)} \left[ \sum_i a_{ij}(\tau_j) p_i + l_j(\tau_j) \right] + \frac{Q_j(\tau_j)}{\pi_j(\tau_j) x_j(\mathbf{p}, \theta)} .$$

It is obvious that  $W_j^{\tau_j}(\mathbf{p}, \theta)$  is positive and continuous in  $\theta$  ( $\theta \in M$ ), because  $x(\mathbf{p}, \theta)$  is continuous in  $\theta$  (see equ. (16)).

Define

$$\mu_j^{\tau_j} \equiv \frac{W_j^{\tau_j}(\mathbf{p}, \theta)}{\sum_{\tau_j \in T_j} W_j^{\tau_j}(\mathbf{p}, \theta)} \quad (\tau_j \in T_j, j=1, \dots, n). \quad (21)$$

Again  $\mu_j^{\tau_j}(\mathbf{p}, \theta)$  ( $\tau_j \in T_j$ ) is a continuous in  $\theta$  ( $\theta \in M$ ) and  $\prod_{\tau_j \in T_j} \mu_j^{\tau_j}(\mathbf{p}, \theta)$  is a mapping from

the compact set  $M$  into the compact set  $M_j$ . Therefore  $\prod_{j=1}^n \prod_{\tau_j \in T_j} \mu_j^{\tau_j}(\mathbf{p}, \theta)$  is a continuous mapping from the compact set  $M$  into itself. Applying the Brouwer's fixed point theorem, we get

$$\theta_j^*(\tau_j) = \frac{W_j^{\tau_j}(\mathbf{p}, \theta^*)}{\sum_{\tau_j \in T_j} W_j^{\tau_j}(\mathbf{p}, \theta^*)} \quad (\tau_j \in T_j, j=1, \dots, n)$$

$$\gamma_j(\tau_j) \equiv \frac{\theta_j^*(\tau_j)}{W_j^{\tau_j}(\mathbf{p}, \theta^*)} = \frac{1}{\sum_{\tau_j \in T_j} W_j^{\tau_j}(\mathbf{p}, \theta^*)} \equiv \gamma_j(\mathbf{p}, \theta^*).$$

This shows, as was to be proved, that there is the set of relative shares  $\{\theta_j^*(\tau_j) \mid \tau_j \in T_j, j=1, \dots, n\}$  which yields the equal rate of returns  $\gamma_j(\mathbf{p}, \theta^*)$  within the same industry  $j$ , although the expected profits differ among firms even in the same industry, reflecting the magnitudes of the existing capital stocks.

In order to obtain some relative shares which bring forth the equal rate of returns over all the industries, we must add some further assumptions on existing capital stocks, which is beyond the scope of this paper. As a matter of fact, it may be plausible to suppose that some capitalist will draw his capital from a less profitable sector and try to invest it to some other sector in which more profit is expected.

But it may result in vain because of some barriers in the sector in which he tries a new entry. Or he may succeed in the entry; this may, however, cause the over-supplies ending with price reduction and higher profit will be unobtainable. What will he do next? He may again try to move and seek for a better profit; he may try to draw some portion of the capital stock and invest a new sector or he may stay and continue to produce in the sector, taking into consideration he learned before.

As a brief consideration above shows that the capital stocks in every firm are the results of many economic factors, we can not obtain the required relative shares unless we have in dynamic framework some concrete relations which regulate the movement of capital stocks.

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