Statistical Indicators of the Movement of the Position of Spatial Distribution of Population: A Statistical Analysis of Regional Data of Population*

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1 Introduction

Fields of statistical analysis of population will be able to be divided roughly into two parts, one is the statistical analysis of size of population, in which the change of the total number of human beings, the demographic structures of birth and death, etc. are treated, and the other is that of distribution of population, in which measuring the central position of distribution of population, concentration or dispersion of population, etc. are discussed.[1]

In this paper, problems which belong to the latter, measuring the position of population distribution and its movement, are examined.

Center of population, median point and other indicators of the position of population distribution have been already defined.[2] But, the characteristics of them have not been sufficiently discussed so that these statistical indicators are not systematically grouped. And the indicators of the change or movement of the position of spatial distribution of population have not utterly been discussed.

In this place, the characteristics of the indicators of the position of population distrubition will be reconsidered, and new statistical indicators of the movement of the position of spatial distribution of population will be studied.

Incidentally, the statistical indicators of the position of the population distribution have been discussed in the field of economic geography as well as population statistics. In economic geography which treats the location problem,[3] the positions of such indicators are very important spots to be discussed.

2 Characteristics of the Indicators of the Position of Spatial Distribution of Population

In demography, usually, "the center of population" is used as the indicator of the position of population distribution. The location of this indicator is expressed by the co-ordinates X and Y, and the co-ordinates of the center of population $\bar{X}$ and $\bar{Y}$ are, as well known, calculated by the equations:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (2.1.1)$$

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad (2.1.2)$$

where X_i and Y_i are the position of the ith persons on the X and Y axes, respectively (see Figure 1).

These are the arithmetic means of the positions of persons measured by the X and Y axes.

Before 1930, unfortunately, a part of students believed that if we measured the sum total of distances between persons and a point (d) as shown in the Figure 2, and tried to find a point which

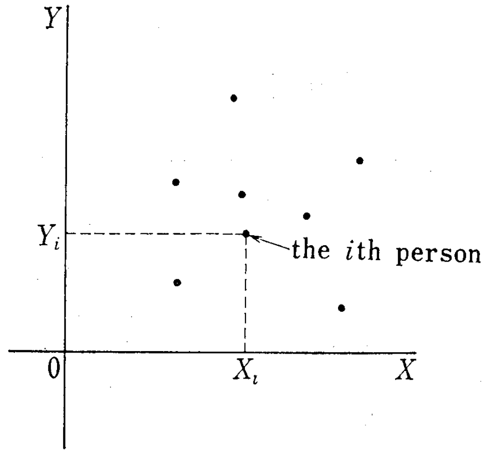

Figure 1 The position of the ith person expressed by the X and Y axes

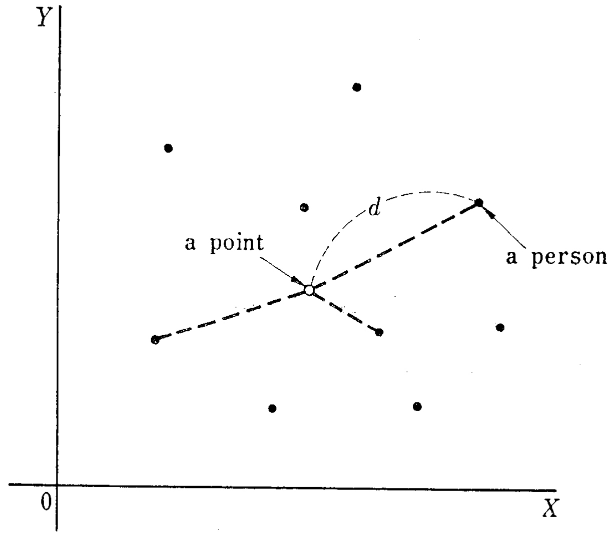

Figure 2 The distance between a person and a point

made the sum total minimum, we found that the point was the center of population, without any deep examination.

In 1930, Eells pointed out that if we measured the sum total of distances between persons and a point, and tried to find a point which made the sum total minimum, we found that the point was not the center of population, but it was another point. The latter of the two points may be called "population center" to distinguish this point from the center of population.[4] Since then, the method of obtaining the population center had been a great question. Fortunately, the mathematical method to determine the position of the population center was found by Kuhn and Keunne in 1962.[5]

A common characteristic of the two points, the center of population and population center, is the minimization of the value related to distance. For the center of population, the total value of squares of the distances is minimized, and for the population center, the sum total of the distances is minimized.

If we denote the distance between the ith person ($i=1, 2, \ldots, n$) and a point by d_i, the value Q^2 is minimized for the center of population, and the Q is minimized for the population center, where

$$Q^2 = \sum_{i=1}^{n} d_i^2 \tag{2.2.1}$$

$$Q = \sum_{i=1}^{n} d_i \tag{2.2.2}$$

Therefore, these points are regarded as the statistical indicators which belong to so called the indicator obtained by the "minimization principle."

We have another indicator obtained by this principle. The indicator is the median point. The median point is the intersection of two median lines. A median line divides the number of persons in the area questioned into two groups which have the same number of persons (see Figure 3). At first sight, median point is not regarded as the indicator obtained by minimization principle, but as the indicator obtained by the equalization principle, because this point is obtained by the median lines which have the characteristics mentioned above. However, this is also the indicator obtained by the minimization principle, because if we measure the sum total of the distances between persons and a line, and try to find a line which makes the sum total minimum, we find that the line is the median line. This characteristic of median line has been already found from the characteristic of the median itself.[6] Therefore, the median point is regarded as the indicator of the position of spatial distribution of population obtained by two principle, the equalization principle and the minimization principle.

We can point out that the median point is the indicator obtained by the minimization principle from another reason. If we measure the sum total of squares of the number of persons found in the two parts of the area questioned surrounding a line (n_1 and n_2) and try to find a line which makes the sum total minimum, we find that the line is the median line. Mathematically saying, the value X^2 defined by the equation:

$$X^2 = n_1^2 + n_2^2 \tag{2.3}$$

is minimized when the line is the median line.[7]

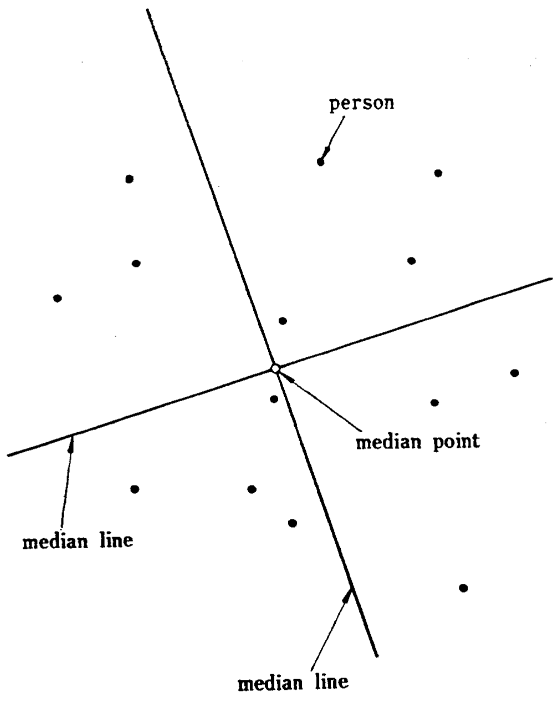

Figure 3 Median lines and median point

This fact has not been mentioned, but this may be also important for regarding the median point as a kind of the indicator obtained by the minimization principle.

We have the other indicator which is obtained by the minimization principle. It is the "average position." The average position was proposed by D. S. Neft for the measurement of the position of social phenomena in space.[8]

When the average position is applied to the measurement of the position of population distribution, it becomes the position of a spot S which minimizes the following value $\sqrt[n]{M_n'}$:

$$\sqrt[n]{M_n'} = \frac{\sqrt[n]{\sum_{i=1}^{m} P_i r_{is}^n}}{P} \tag{2.4}$$

where P_i is the population at the ith spot ($i=1, 2, \ldots, m$), r_{is} the distance between the ith spot and the spot S measured along the great circle of the earth, and P the total population observed $\sum_{i=1}^{m} P_i$ (Figure 4).

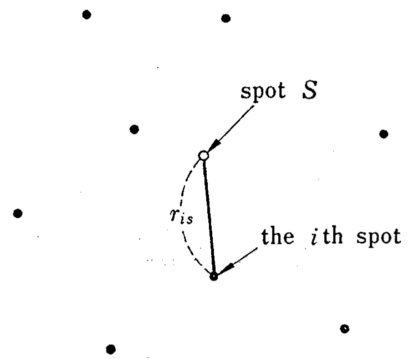

Figure 4 The spot S, the ith spot and the distance r_{is} for the Neft's "average position"

We can say that this indicator should be also regarded as an indicator obtained by minimization principle.

In general, if we call the statistical indicator of the position of population distribution obtained by minimizing the sum total of the nth power of some value V obtained from the observation of distribution of persons "the nth center of population related to V," then the center of population can be called the 2nd center of population related to distance, population center the 1st center of population related to distance, the Neft's average position the nth center of population related to distance, and median point the 2nd center of population related to the number of persons as well as the 1st center of population related to distance.

Based on the discussions stated above, we can say that we have a group of statistical indicators obtained by the minimization principle, although we may have indicators obtained by other principles like the equalization principle.

3 The *n*th Center of Population Related to S_h

If we defined the value S_h as follows, we can easily identify the two kind of indicators of the position of population, the center of population and population center.

$$S_h = \sum_{i=1}^{n} P_i \{ \sqrt{(x_i-x)^2+(y_i-y)^2} \}^h \tag{3.1}$$

where P_i is the population at the ith point, x_i and y_i ($i=1, 2, \ldots, n$) are the co-ordinates x and y

of the ith point, respectively, when we give the X and Y axes to the surface of the ground, and x and y are the co-ordinates x and y of a point arbitrally determined, respectively.

The center of population is the point whose position is expressed by the co-ordinates x and y which minimize the S_2, and the population center is the point whose position is expressed by the co-ordinates x and y which minimize the S_1.

Therefore, the center of population and the population center can be called the 2nd center and the 1st center of population related to S_k, respectively.

4 Measuring the Change of the Position of the Indicators of Spatial Distribution of Population

4. 1 *Net size of movement of population*

When the position of the distribution of population changes, the location of the indicator of the position will also change. And we can measure the degree of the movement of the position of distribution of population easily by measuring the distance of the movement of the nth center of population or other indicator of the position of population distribution.

But, we can express the degree of the movement of the position of spatial distribution of population by another method. In general, the degree of the movement will be able to be expressed by the product of the distance of movement of the position of the indicator of population distribution ($D\Gamma$) and population ($\bar{P}$). Here, the product is called "net size of movement of population ($\delta\Gamma_P$)." The product is defined mathematically as follows.

$$\delta\Gamma_P = \frac{(D\Gamma)\bar{P}}{T} \tag{4.1.1}$$

or

$$\delta\Gamma_P = (\delta\Gamma)\bar{P} \tag{4.1.2}$$

where $\delta\Gamma_P$ is the net size of movement of population during the period of time of observation, $D\Gamma$ the distance of movement of the location of the indicator of the population distribution during the period, $\bar{P}$ the average population calculated by the populations during the period, T the length of the period, and $\delta\Gamma$ the distance of movement of the location of the indicator of the population distribution per one unit of time during the period ($D\Gamma$ T).

If all the person questioned move by $D\Gamma$ in some direction during the period of time observed, the location of the indicator of the position of population distribution also moves by $D\Gamma$ in the same direction. Then, the $\delta\Gamma$ is regarded as the total length of movement of population per one unit of time.

But, the actual movement of persons appears in every direction. Some persons go west and others go east. Therefore, $\delta\Gamma_P$ is not always regarded as the estimate of the total length of the movement of population per one unit of time interval M, that is to say, "gross size of movement of population." The gross size of movement of population M will be much larger than the total length of the movement of population. The latter should be called the net size of movement of population as called above.

The reason why we call the indicator $\delta\Gamma_P$ "'net' size of movement of population" is that the $\delta\Gamma_P$ is regarded as the 'total' length of the movement of persons which affects the movement of location of the indicator of the position of population distribution, in other words, 'net' affect of the movement of persons to the movement of the indicator of the positoin of population. For example if only two persons moved and exchanged their positions, their movements do not affect to the movement of location of the indicator of the position of population distribution. Therefore, in this case, $\delta\Gamma_P$ is 0. But, total length of the actual movement of population is not zero, because the two persons moved and exchanged their positions.

If we have statistics of interregional or interzonal migration of population shown by the origin-destination table, we will be able to estimate the gross size of movement of population M by the statistics in the origin and destination table. And we will be able to find the quantitative relationship between the net size of movement of population calculated by some indicator of the position of population distribution $\delta\Gamma_P$ and the gross size of movement of population M.

When we measure the gross size of movement of population, we must consider the movement of the immigrants and emigrants of the area questioned. If the numbers of the immigrants and emigrants of the area questioned are relatively small, we can neglect the length of their movements. But, if the numbers of them are relatively large, of course, we can not neglect the length of their movements.[9]

Incidentally, the gross size of movement of population has close relation with economic problem.

A rough estimate the total cost expended for the movement of population in the area questioned C will be given by the product of the gross size of movement of population M and the cost for the movement of a person by one unit of distance c, that is to say,

$$C = cM \tag{4.2}$$

Therefore, the M is a good indicator of the C. And the ratio E_M defined by

$$E_M = \frac{\delta\Gamma_P}{M} \tag{4.3}$$

will show the degree of the effect of the gross size of movement of population M on the movement of the location of the indicator of the position of population distribution, and it will be regarded as the indicator of the regularity of movement of population. If most of the persons observed moves in the same direction, the value of E_M will become large and approach to 1.

4.2 *Standardized movement of the location of the indicator of the position of spatial distribution of population*

In demography, some statistics are often standardized by the structure of population.[10] For the movement of the location of the indicator of the position of spatial distribution of population, we can also consider a kind of standardization.

According to the definition of $\delta\Gamma_P$, we can find the $\delta\Gamma$ (the distance of movement of location of the indicator of the population distribution per one unit of time during the period of time of observation) by the following equation:

$$\delta\Gamma = \frac{\delta\Gamma_P}{\bar{P}} \tag{4.4}$$

Therefore, if we use a standard population P^S instead of $\bar{P}$, then we will be able to obtain a new value, "standardized movement of population" $\delta\Gamma^{(S)}$ which is defined by the equation:

$$\delta\Gamma^{(S)} = \frac{\delta\Gamma_P}{P^S} \tag{4.5}$$

This value is regarded as the distance of movement of the location of the indicator of the position of spatial distribution of population when the population observed is unchanged.

By this value, the degree of the movement of the position of spatial distribution will be expressed in the form of length, while $\delta\Gamma_P$ expresses the degree by the form of product of length and mass, that is to say, the product of 'the distance of the movement of the location of the indicator of the position of population' and 'population.'

5 The Change of the Position of Spatial Distribution of Population in Japan, 1920–1975

The locations of the two indicators of the population of spatial distribution of population, the "center of population" and "population center" from 1920 to 1975 in Japan are shown in Table 1. These values are calculated by the populations of all the prefectures in Japan except Okinawa,[11] and the locations of the seats of prefectural governments. Here, it is assumed that all the persons of each prefecture live at the seat of the prefectural government. And the locations of these indicators are shown by the unit of the "degree" of latitude and longitude. (The "minute" is not used here. Therefore, for example, longitude 136°42′ East is expressed by longitude 136.70° East.)

Table 1 The location of the center of popolation and population center of Japan 1920–1975

Year	C.P.		P.C.	
	LA(N)	LO(E)	LA(N)	LO(E)
1920	35.68°	136.53°	35.40°	136.76°
1925	35.68	136.57	35.40	136.77
1930	35.70	136.62	35.40	136.77
1935	35.70	136.66	35.39	136.78
1940	35.70	136.70	35.38	136.82
1945	35.81	136.69	35.44	136.82
1950	35.78	136.70	35.42	136.85
1955	35.78	136.74	35.41	136.90
1960	35.78	136.82	35.41	136.97
1965	35.77	136.93	35.41	137.05
1970	35.76	137.01	35.42	137.14
1975	35.74	137.06	35.44	137.21

Notes: C.P.: center of population
P.C.: population center
LA(N): north latitude
LO(E): east longitude

The locations of the two indicators are approximately in latitude 35° North and in longitude 137° East and are going east (see Figures 5 and 6).

The distance of the movement of the two indicators, the center of population and population center are shown in Table 2. The distance of the movement per unit of time of the indicators $\delta\Gamma$ is calculated by the following formula.

$$\delta\Gamma = \sqrt{\{\delta\Gamma(N)\}^2+\{\delta\Gamma(E)\}^2} \tag{5.1}$$

where $\delta\Gamma(N)$ and $\delta\Gamma(E)$ are the distances of the movement per one unit of time of the indicators to the directions of North and East, respectively. And the net size of movement of population and standardized movement of population obtained by the two indicators are shown in Table 3.

The values in Tables 2 and 3 are also shown by the unit of the degree of latitude and longitude.

$\delta\Gamma$ and $\delta\Gamma_P$ or $\delta\Gamma^{(S)}$ are gradually increasing. And they became large in the Second World

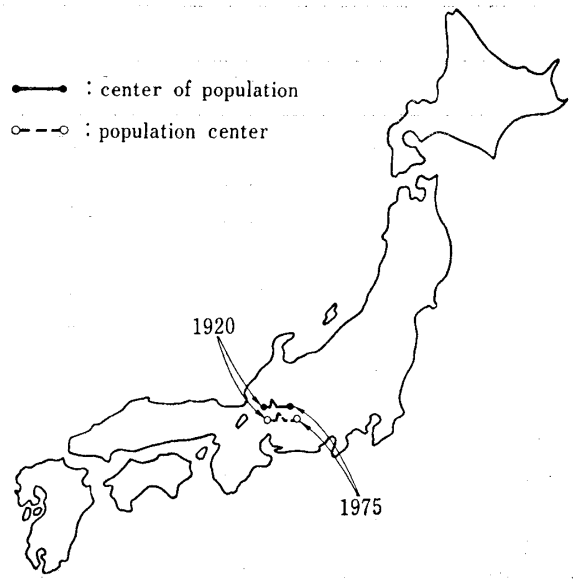

Figure 5 The locations of the center of population and population center (1)

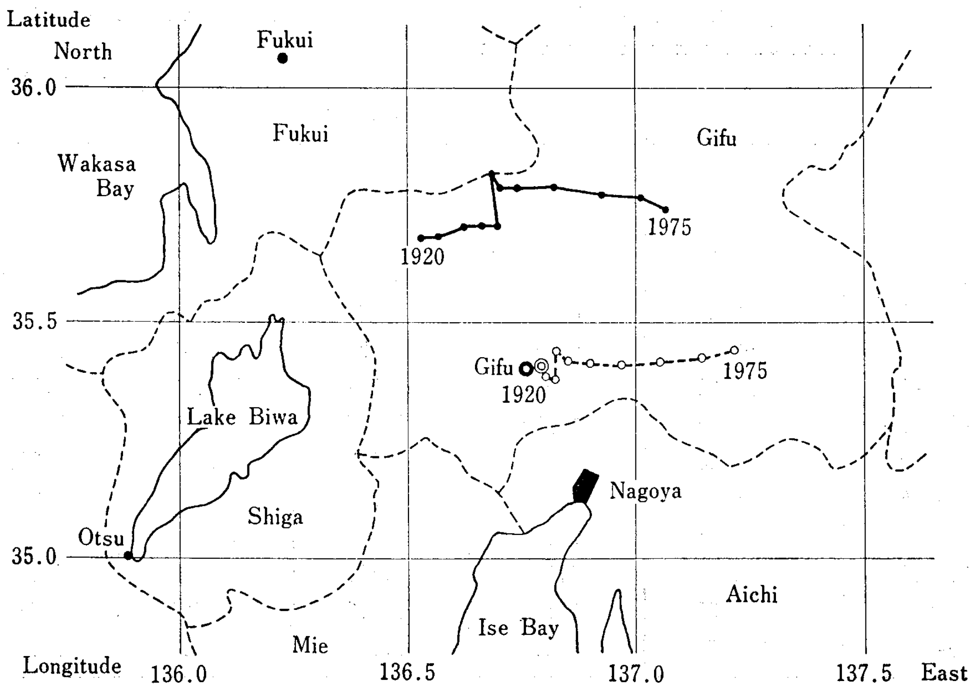

Figure 6 The locations of the center of population and population center (2)

Table 2 The distance of the movement of the location of the indicators, the center of population and population center

(0.001 degree/year)

Period	C.P.			P.C.		
	$\delta\Gamma(N)$	$\delta\Gamma(E)$	$\delta\Gamma$	$\delta\Gamma(N)$	$\delta\Gamma(E)$	$\delta\Gamma$
1920–'25	0°	8°	8.0°	0°	2°	2.0°
1925–'30	4	10	10.8	0	0	0.0
1930–'35	0	8	8.0	−2	2	2.8
1935–'40	0	8	8.0	−2	8	8.2
1940–'45	22	−2	22.1	12	0	12.0
1945–'50	15	2	6.3	−4	6	7.2
1950–'55	0	8	8.0	−2	10	10.2
1955–'60	0	16	16.0	0	14	14.0
1960–'65	−2	22	22.1	0	16	16.0
1965–'70	−2	16	16.1	2	18	18.1
1970–'75	4	10	10.8	4	14	14.6

Notes: $\delta\Gamma(N)$: the distance of the movement of location of the indicators in the northernly direction
$\delta\Gamma(E)$: the distance of the movement of location of the indicators in the easternly direction

Table 3 The net size of movement of population and standardized movement of population $\delta\Gamma_P$ obtained by the two indicators, the center of population and population center

(million persons × degree/year (for $\delta\Gamma_P$))
(degree/year (for $\delta\Gamma^{(S)}$))

Period	$\bar{P}$ (million)	$\delta\Gamma_P$		$\delta\Gamma^{(S)}$	
		C.P.	P.C.	C.P.	P.C.
1920–'25	57.29	0.458	0.115	0.0080	0.0020
1925–'30	61.53	0.664	0.000	0.0116	0.0000
1930–'35	66.27	0.530	0.186	0.0093	0.0032
1935–'40	70.60	0.565	0.589	0.0099	0.0103
1940–'45	72.27	1.598	0.868	0.0279	0.0152
1945–'50	77.60	0.490	0.560	0.0086	0.0098
1950–'55	86.24	0.690	0.879	0.0120	0.0153
1955–'60	91.35	1.461	1.278	0.0255	0.0223
1960–'65	95.85	2.117	1.533	0.0370	0.0268
1965–'70	101.00	1.628	1.830	0.0284	0.0319
1970–'75	107.31	1.159	1.567	0.0202	0.0274

Note: P^S for the calculation of $\delta\Gamma^{(S)}$ is 57.29 millions, the average population from 1920 to 1925 (the arithmetic mean of the populations in 1920 and 1925). The average population for each period of time is added to this table for reference.

War (from 1940 to 1945), because people dispersed from urban regions to rural regions at that time and the position of population distribution was largely changed.

6 The Relationship between the Movement of the Indicators of the Position of Spatial Distribution of Population and Concentration of Population

Hoover proposed the index of population concentration $\varDelta$ to express the degree of spatial concentration of population in the area observed. Hoover's index of population concentration $\varDelta$ is defined by

$$\varDelta = \frac{1}{2}\Sigma\left|\frac{P_i}{P}-\frac{S_i}{S}\right| \tag{6.1}$$

where P_i and S_i are the population and area of the ith region, respectively, and P and S the total population and area of all the regions observed.[12] When the degree of concentration of population becomes high, the $\varDelta$ becomes also large. And $\varDelta$ appears between 0 and 1.

For measuring the velocity of the change of the value of $\varDelta$, I defined $\delta\varDelta$. $\delta\varDelta$ is defined by the following equation:

$$\delta\varDelta = \varDelta_1-\varDelta_0 \tag{6.2}$$

where $\varDelta_0$ and $\varDelta_1$ are the values of the $\varDelta$'s at the beginning of one period of time and at the beginning of the next period of time, respectively.

It is very interesting that we can find a close relationship between the value of the change of the Hoover's concentration coefficient of population $\delta\varDelta$ (see Table 4) and the $\delta\Gamma$ and $\delta\Gamma_P$ (see Figure 7). The graphs show that when the $\delta\varDelta$ or the absolute value of $\delta\varDelta$ are large, the $\delta\Gamma$ and $\delta\Gamma_P$ become also large.

According to the fact, we can say that, in Japan, the position of spatial distribution of population has had close relation with the concentration of population to the eastern part of the coast of Pacific Ocean from Osaka to Tokyo.

Table 4 Hoover's concentration coefficient of population $\varDelta$ and $\delta\varDelta$

1. $\varDelta$

Year	$\varDelta$
1920	29.68%
1925	30.21
1930	30.67
1935	31.65
1940	33.17
1945	28.14
1950	30.24
1955	31.43
1960	33.33
1965	36.06
1970	38.53
1975	39.87

2. $\delta\varDelta$

Period	$\delta\varDelta$
1920–'25	0.106%
1925–'30	0.092
1930–'35	0.196
1935–'40	0.304
1940–'45	−1.006
1945–'50	0.420
1950–'55	0.238
1955–'60	0.380
1960–'65	0.546
1965–'70	0.494
1970–'75	0.268

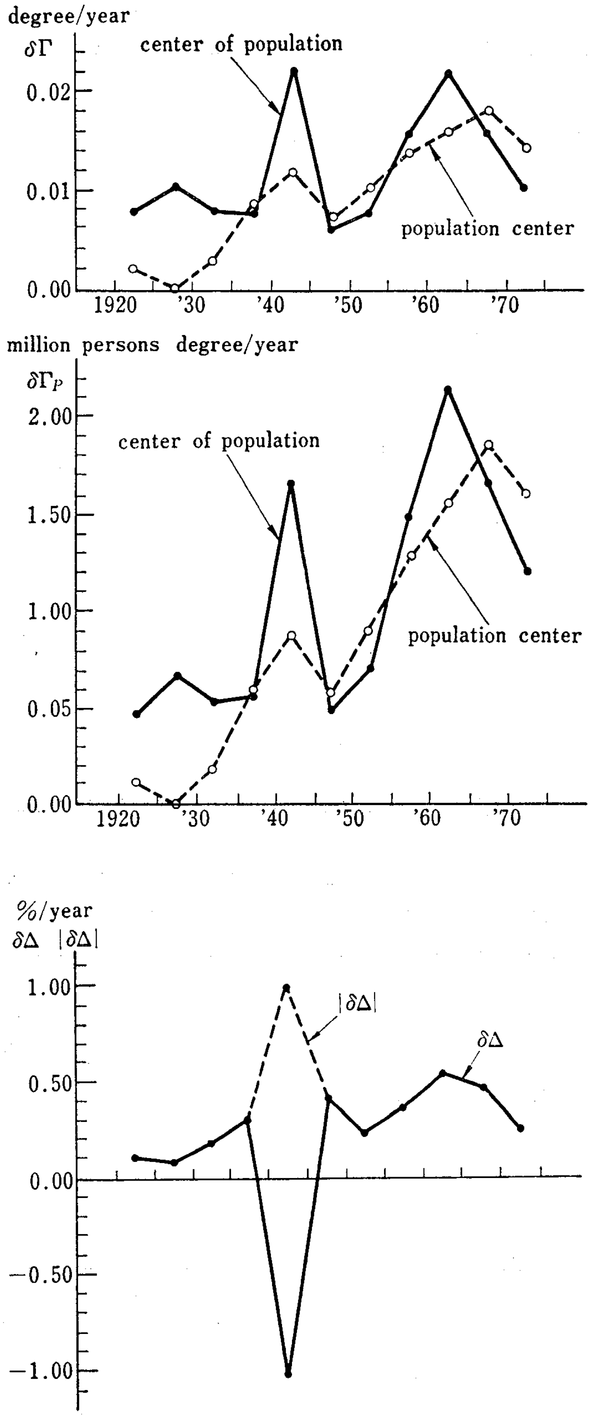

Figure 7 The comparison between $\delta\Gamma$, $\delta\Gamma_P$ and $\delta\Delta$

7 Conclusion

In this paper, the indicators of the position of spatial distribution of population and its movement were discussed.

First of all, the existence of a group of the indicators of the position of spatial distribution of population obtained by minimization principle was pointed out.

Secondly, the gross and net size of movement of population were defined and discussed.

Thirdly, the standardized movement of population was proposed and examined.

Lastly, the change of the position of spatial distribution of population in Japan from 1920 to 1975 was actually measured by the net size of movement of population. And it was found that in Japan, the change of the position of population distribution has close relation with the concentration of population to the eastern part of the coast of Pacific Ocean from Osaka to Tokyo.

Notes

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4 Harold W. Kuhn and Robert E. Kuenne who found the method to obtain the position of this point gave the name "population center" when they discussed this point.

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The approximate values showing the position of the population center obtained at the kth step of the calculation, $\bar{X}^{(k)}$ and $\bar{Y}^{(k)}$ in the Kuhn and Kuenne's method are shown by the following equations:

$$\bar{X}^{(k)} = \frac{\sum_{i=1}^{m} P_i^{(k-1)} X_i}{\sum_{i=1}^{m} P_i^{(k-1)}}$$

$$\bar{Y}^{(k)} = \frac{\sum_{i=1}^{m} P_i^{(k-1)} Y_i}{\sum_{i=1}^{m} P_i^{(k-1)}}$$

$$P_i^{(k-1)} = \frac{P_i}{d_i^{(k-1)}}$$

$$d_i^{(k-1)} = \sqrt{(X_i-\bar{X}^{(k-1)})^2+(Y_i-\bar{Y}^{(k-1)})^2}$$

where

$\bar{X}^{(k)}$ and $\bar{Y}^{(k)}$: the positions of the population center on the X and Y axes obtained by the kth step of calculation, respectively,

P_i: the population at the ith spot ($i=1, 2, \ldots, m$)

X_i and Y_i: the positions of the ith spot on the X and Y axes, respectively,

$P_i^{(k-1)}$ when $k=1$: P_i

The $\bar{X}^{(k)}$ and $\bar{Y}^{(k)}$ are regarded as the positions of population center on the X and Y axes, when

$$\bar{X}^{(k)} \fallingdotseq \bar{X}^{(k-1)}$$
$$\bar{Y}^{(k)} \fallingdotseq \bar{Y}^{(k-1)}$$

by increasing the number of step k.

6 Flaskämper, Paul: *op. cit.*, S. 118.

7 When $n_1=n_2$, the S becomes minimum.

Proof.

If we minimize the S subject to the restriction: $n_1+n_2=n$, the condition written by the following equations must be satisfied.

$$\frac{\partial}{\partial n_1}\{n_1^2+n_2^2-\lambda(n_1+n_2-n)\} = 0$$

$$\frac{\partial}{\partial n_2}\{n_1^2+n_2^2-\lambda(n_1+n_2-n)\} = 0$$

where n is the total number of persons and λ is the Lagrangian multiplier.

Then, we can obtain the result that

$$n_1 = n_2 = \frac{\lambda}{2}$$

Q.E.D.

Incidentally, when the total number of persons observed is even number, the number of persons in one part among the two parts made by median line is $(n_1+n_2)/2$, on the other hand, when the total number of persons observed, the number of persons in one part among the two parts is $(n_1+n_2-1)/2$, because in the case that the total number of persons observed is odd number, one person is always on the median line and the number of persons who are not on the median line is n_1+n_2-1. (The n_1+n_2-1 persons devided in two parts by a median line.)

The origin-destination table for
inter-zonal migration of population

O \ D	1	2	…	r	…	$r+m-1$	…	Z
1	$P_{1\ 1}$	$P_{1\ 2}$	…	$P_{1\ r}$	…	$P_{1\ r+m-1}$	…	$P_{1\ Z}$
2	$P_{2\ 1}$	$P_{2\ 2}$	…	$P_{2\ r}$	…	$P_{2\ r+m-1}$	…	$P_{2\ Z}$
⋮	⋮	⋮		⋮		⋮		⋮
r	$P_{r\ 1}$	$P_{r\ 2}$	…	$P_{r\ r}$	…	$P_{r\ r+m-1}$	…	$P_{r\ Z}$
⋮	⋮	⋮		⋮		⋮		⋮
$r+m-1$	$P_{r+m-1\ 1}$	$P_{r+m-1\ 2}$	…	$P_{r+m-1\ r}$	…	$P_{r+m-1\ r+m-1}$	…	$P_{r+m-1\ Z}$
⋮	⋮	⋮		⋮		⋮		⋮
Z	$P_{Z\ 1}$	$P_{Z\ 2}$	…	$P_{Z\ r}$	…	$P_{Z\ r+m-1}$	…	$P_{Z\ Z}$

8 Neft, David S.: *Statistical Analysis for Areal Distributions*, Philadelphia, Regional Science Research Institute, 1966.

9 If the area considered is devided into Z zones and the area questioned is consists of m zones from the rth to the $(r+m-1)$th zone among them, then the gross size of movement of population of the area questioned M is calculated by the equation:

$$M=\sum_{v=1}^{Z}\sum_{u=r}^{r+m-1}P_{uv}\,d_{uv}+\sum_{u=r}^{r+m-1}\sum_{v=1}^{Z}P_{uv}\,d_{uv}-\sum_{u=r}^{r+m-1}\sum_{u=r}^{r+m-1}P_{uv}\,d_{uv}$$

where P_{uv} is the number of persons who have moved from the uth zone to the vth zone during the time interval observed, and d_{uv} is the distance between the uth and the vth zones.

If the number of persons P_{uv} is represented in the origin-destination table as shown above, then the numbers of persons which have relation with the calculation of the gross size of movement of population appear in m rows from the rth to the $(r+m-1)$th row and m columns from the rth and $(r+m-1)$th column.

10 For example, we have standardized birth rate calculated by the standard population structure by age.

11 Japan, Sorifu, Tokeikyoku (ed.): *Dai 27 kai Nippon Tokei Nenkan*, (*The 27th Statistical Year Book of Japan*), Tokyo, Nippon Tokei Kyokai and Mainichi Shimbun-sha, 1977, pp. 12–14.

12 Duncan, Otis Dudley, Cuzzort, Ray P., Duncan, Beverly: *Statistical Geography*, Illinois, the Free Press of Glenco, 1961, p. 83.

要　　約

鈴木啓祐：「人口の空間的分布の位置の変化に関する統計的指標について：地域的人口資料の統計的解析」『流通経済大学論集』第14巻第1号，1979年，34-47ページ。

この論文の主要な部分は，1978年，パリ大学で開催された日仏セミナー (Franco-Nippono Séminaire, (Colloque Franco-Japonais)) に提出した論文によって構成されている。

ここでは，人口の空間的（場所的）分布の位置（人口中心）とその移動に関する指標について論じた。

まず，第1に，人口分布の位置の指標（人口中心）の中には，『「最小化の原理」によって得られる指標』とみなされる一群の指標が存在すること，そして，h 次の人口中心を

$$S_h=\sum P_i\{\sqrt{(x_i-x)^2+(y_i-y)^2}\}^h$$

で定義される S_h を最小にさせる地点 (x, y) と定義したとき，人口重心は2次の人口中心，人口中心点は1次の人口中心であることを指摘した。

第2に，人口分布の変化，あるいは，人口移動の状態を示すための指標として，人口中心の総移動規模 (gross size of movement of population) および純移動規模 (net size of movement of population) を定義し，これらについて論じた。

第3に，人口中心の移動距離の標準化について論じた。

最後に，実際に，1920年から1975年までのわが国の人口の分布の位置の変化を人口中心——特に，人口重心および人口中心点——の移動量およびその純移動規模によって測定し，これらの大きさが，わが国の太平洋沿岸への人口集中の速度ときわめて密接な関係のあることを見いだした。