Quantitative Indicators for Measuring the Characteristics of the Change of the Geographical Distribution of Population

Keisuke Suzuki

1. Introduction*

We have now many kinds of indicators of the characteristics of spatial or geographical distribution of population at a certain point of time which have been proposed by many students. Some kind of indicators of density, indicators for heterogeneity of population distribution and those for certain point of population distribution are the indicators.[1]

For example, we have density,[2] areality,[3] and proximity[4] as the indicators of the distribution of population, index of population concentration proposed by Hoover,[5] the λ of the Lorenz curve method,[6] location quotient proposed by Florence[7] as the indicators for the heterogeneity of distribution of population, and some kinds of central points of population distribution—center of population, population center, etc.—as the indicators of central point of population distribution.[8]

When we observe the spatial distribution of a population, we can know the profile of the distribution of the population by means of the indicators.

However, if we can observe additionally the characteristics of the change of spatial distribution of population during a period of time quantitatively, we would be able to grasp the profile or structure of the distribution of population more deeply.

But, unfortunately, we have few indicators for the characteristics of the change of spatial distribution of population.

In this paper, I would like to propose the indicators for measuring the characteristics of the change of spatial distribution of population.

2. Indicators

The indicators proposed here are classified into 3 groups:
  I. Indicators for the change of the central point of population distribution
  II. Indicators for the change of the direction of the axis of population distribution, and
  III. Indicators for the change of the pattern of population distribution.
For the first group (Group I), I would like to propose, here, 3 kinds of indicators:
  1. Length of the movement of central point $\delta \Gamma$ which is the distance between 2 central points of population obtained by 2 points of time,
  2. Size of the movement of central point $\delta \Gamma_P$ which is the product of the $\delta \Gamma$ and population P.
    By this indicator, we can show the " size " of the distance of the movement of central point of population weighted by population.[9]
  3. Direction of the movement of central point, W.

The quantities measured by these indicators, $\delta \Gamma$, $\delta \Gamma_P$ and W are shown pictorially by (1) the length of a straight line between the two central points at different points of time, (2) the area of a rectangle which has $\delta \Gamma$ as the length of one side and population P as that of the other side, and (3) the direction of the movement of the central point measured by a basic direction, respectively, as shown in Figs. 1 (1), (2) and (3).

When we obtain the actual values of these indicators, we must determine the central point of
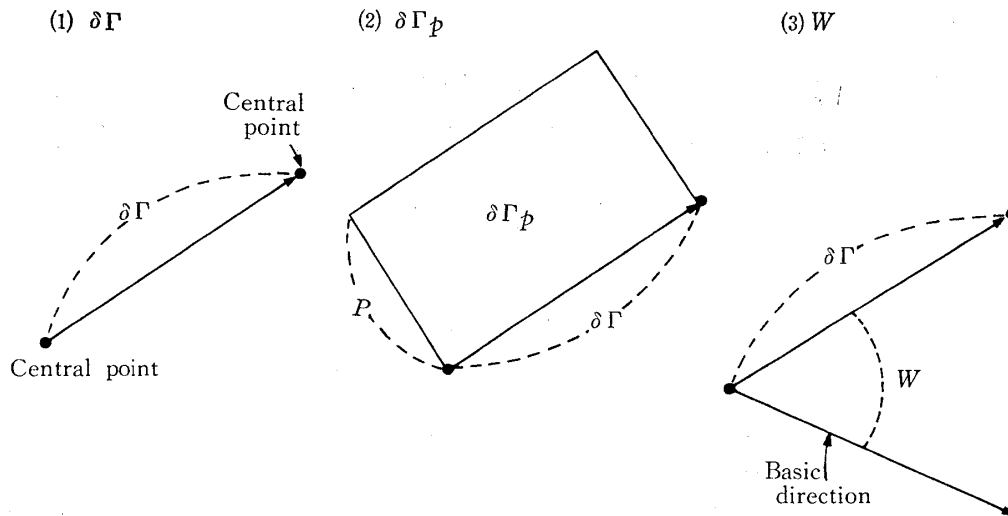
(1) $\delta\Gamma$ (2) $\delta\Gamma_p$ (3) W

Figure 1 Pictorial representation of $\delta\Gamma$, $\delta\Gamma_P$ and W.

distribution of population. In general, we can show the central point by the hth center of population ($h=1, 2, 3, ...$) which is the point whose co-ordinates x and y are those which minimize the value S_h defined by

$$S_h = \sum_{i=1}^{n} P_i \{ \sqrt{(x_i-x)^2+(y_i-y)^2} \}^h \tag{2.1}$$

where P_i ($i=1, 2, 3, ..., n$) is the population in the ith region (R_i) and x_i and y_i are the co-ordinates of the location of the region R_i (Fig. 2).
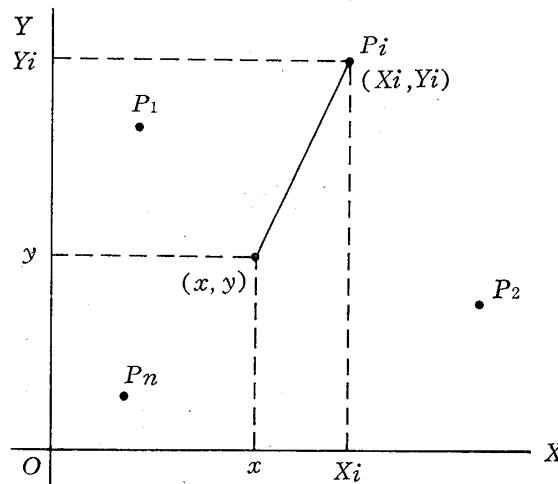

Figure 2 The elements of determination of S_h.

When h is equal to 2, then the central point of population which is obtained by this definition is the center of population, because the center of population can be regarded as the 2nd center of population.

Now we have the mathematical methods to obtain the first and the second center of population: the former is the Kuhn-Kuenne's method[10] and the latter is the method to obtain a center of population.[11]

In the second group (Group II) of the indicators, we can find 2 kinds of indicators:

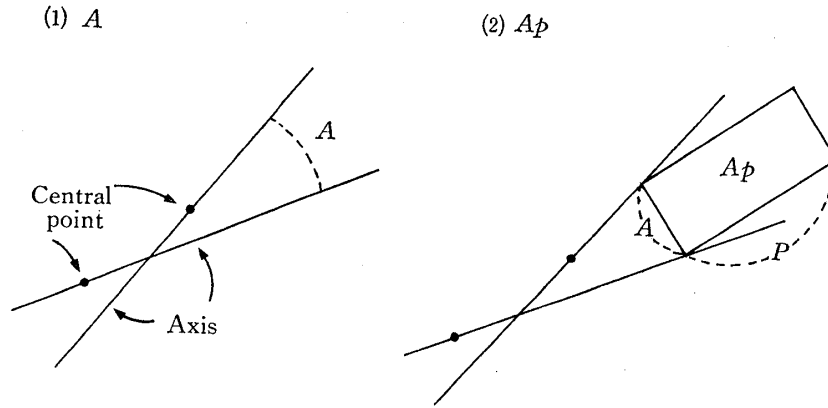   1.   Angle of the change of the direction of axis, A

(1) A (2) Ap

Figure 3   Pictorial representation of A and A_P.

2. Size of the change of the direction of axis, A_P which is the product of A and population P.[12]

These indicators are pictorially shown by Figs. 3 (1) and (2). As shown in Fig. 3, we can show clearly the angle A and the area A_P.

For the third group (Group III) of the indicators, we can give 2 kinds of indicators:

1. Change of degree of population concentration $\delta\varDelta$ which is measured by the difference between 2. Hoover's indices of population concentration obtained at 2 points of time for a population in the regions observed, $\varDelta_t - \varDelta_{t-1}$ where $\varDelta_t$ is, as it is well known, defind by

$$\varDelta_t = \frac{1}{2} \sum_{i=1}^{n} |p_{it} - a_{it}| \qquad (2.2)$$

in this equation, p_{it} is the actual ratio of the population of the jth region (R_i) to the total population of all the regions questioned at time t, and a_{it} is the actual ratio of the area of the jth region to the total area of all the regions questioned.

2. Change of the pattern of population $\varDelta_D$ which is the value defined by

$$\varDelta_D = \frac{1}{2} \sum_{i=1}^{n} |p_i - \hat{p}_i| \qquad (2.3)$$

where p_i is the ratio of the actual population of the ith region (p_i) to the total population of all the regions questioned (P) at time t, namely p_{it} and $\hat{p}_i$ is the ratio of calculated
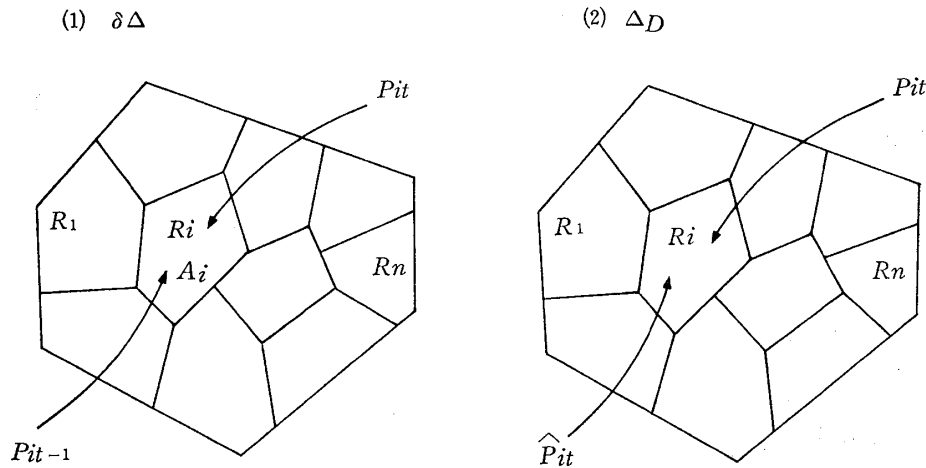
(1) $\delta\triangle$ (2) $\triangle_D$

Figure 4   Factors of for calculation of $\delta\varDelta$ and $\varDelta_D$.

population of the ith region ($\hat{p}_i$) to the total population (P) at time t under the supposition that interregional migration is not found during the period from time $t-1$ to time t. For practical calculation, I regarded p_{it-1} as $\hat{p}_i$.

In Figs. 4. (1) and (2), we can see the factors for calculation of these indicators.

3. Examples of Application of Indicators

I would like to show some examples of measuring the characteristics of the change of distribution of population by the indicators stated above.

First of all, I can show an example of measuring $\delta\Gamma$ and $\delta\Gamma_P$ for the movement of the center of population in Japan during 5 years. Fig. 5 shows the $\delta\Gamma$ and $\delta\Gamma_P$ from 1920 to 1975 which are calculated by the center of population shown in Fig. 6.[13]
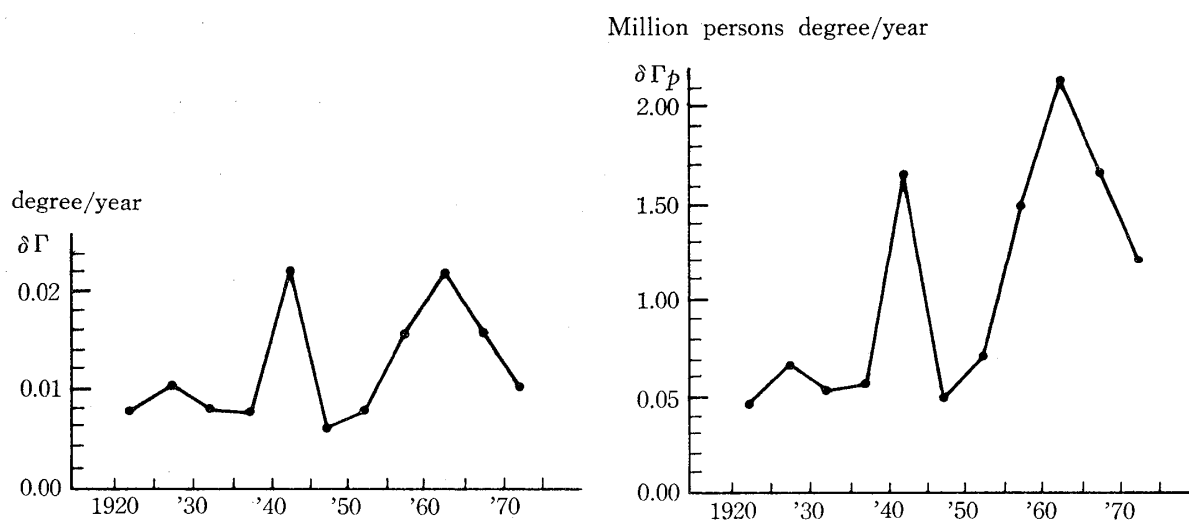

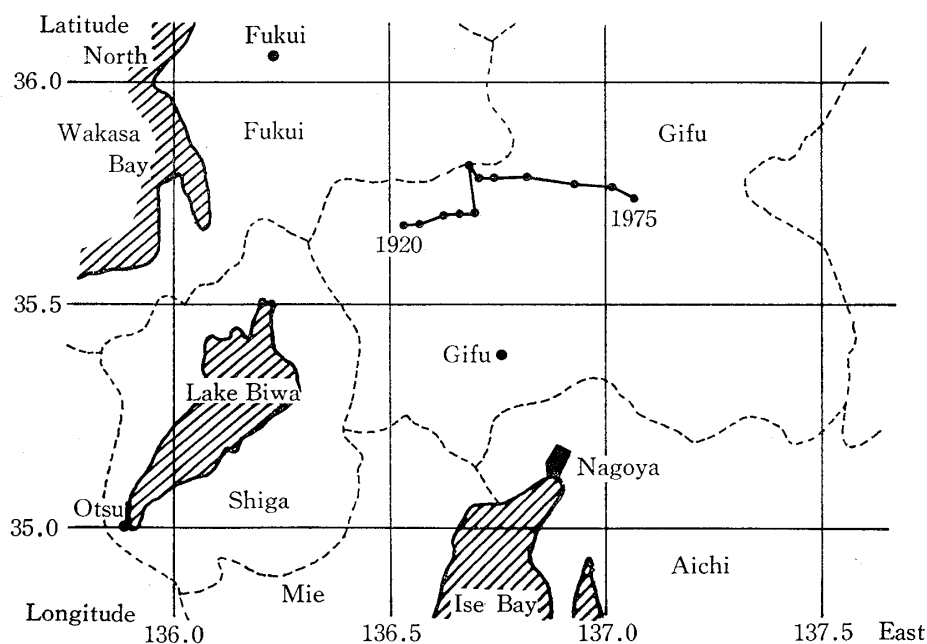Figure 5 $\delta\Gamma$ and $\delta\Gamma_P$ calculated by the center of population of Japan (1920-1975).

Figure 6 The locations of the center of population.

The values of $\delta\Gamma$ for the periods from 1935 to 1940 and from 1950 to 1955 are approximately equal to each other, but those of $\delta\Gamma_P$ for the periods are not equal to each other. From this result, we can find that the " length " of the movement of the center of population $\delta\Gamma$ of Japan for the period from 1950 to 1955 was equal to that for the period from 1935 to 1940, while the " size " of movement of the central point of population $\delta\Gamma_P$ of this country for the period from 1950 to 1955 was larger than that for the period from 1935 to 1940. We can also find the similar fact. The " length " from 1960 to 1965 was approximately equal to that for the period from 1940 to 1945, while the " size " for the period from 1960 to 1965 was larger than that for the period from 1940 to 1945.

Secondly, I would like to show an example of actual observation of the direction of the movement of the central point W.

Figure 7 shows the movement of the center of population for cohorts for the period from 1965 to 1970.[14] And Fig. 8 shows the frequency distribution of direction of the movement of the central point of population W obtained from Fig. 7. The angle of the movement of the center of population for cohorts W was measured counterclockwise from the basic direction to the direction of the line by which the movement of the center of population for cohorts was shown.

In this case, the direction of the line obtained by combining the center of population of a cohort for 1965 and the center of net product for 1970 was regarded as the basic direction for measuring the W of the cohort. And the directions of the basic line and the line showing the movement of the center of population for cohorts are drawn under the supposition that the length of the 1 degree of latitude and that of longitude are exactly equal to each other.[15]

By this observation, we could find that most of the directions of the movement of the central point of population of cohort W for the period from 1965 to 1970 distributed around the direction to the center of net product in 1970.

Thirdly, I can show the example of the angle and size of the change of the direction of the axis A and A_P. Figure 9 shows the axis of the population distribution in 1960 in the regions (namely cities and prefectures) surrounding Osaka. The angle A for population and that for the population of the first sector of industry from 1960 to 1970 are shown in Fig. 10, where the axis was determined by the principle component analysis and it is the first axis of the analysis.[16]

From this result, we can say that the change of the axis of distribution of population (which is 4°) and that of the population of the first sector of industry (which is $-5°$) are approximately equal to each other, apart from the direction of the change of the axis (Exactly saying, the former is slightly smaller than the latter.). The direction of the movement of the axis of population was clockwise, while that of the population of the first sector of industry was counterclockwise.

It is very interesting to find the fact that absolute value of the size of the change of the axis of distribution of population A_P obtained by using the population in 1965 (which was 58.8 million person-degrees) was very large as compared with that of the population of the first sector of industry (which was 4.5 million person-degrees),[17] because the population in 1965 was so large that A_P of population became larger than that of the population of the first sector of industry.

Last of all, I would like to show an example of measuring $\delta\Delta$ and Δ_D.

Figure 11 is the $\delta\Delta$ obtained by the prefectural data of Japan which is calculated by the Hoover's concentration coefficient of population of Japan[18] which is shown in Table 1. We find a large decrease of $\delta\Delta$ in the period from 1940 to 1945, in the graph in Fig. 11. From this decrease, we can clearly know that in the second world war, the population of Japan dispersed abruptly. The strong change of the degree of population concentration is also suggested by the change of the value of $|\delta\Delta|$.

Figure 12 shows the Δ_D for the cohorts from 1965 to 1970 and from 1970 to 1975 in Japan.[19] From this figure, we can find that the results for both periods of time are very similar to each other and the Δ_D's for the cohorts from 10 to 25 years of age are very high, and on the contrary, the Δ_D for the cohort from 25 to 30 years of age is very low.
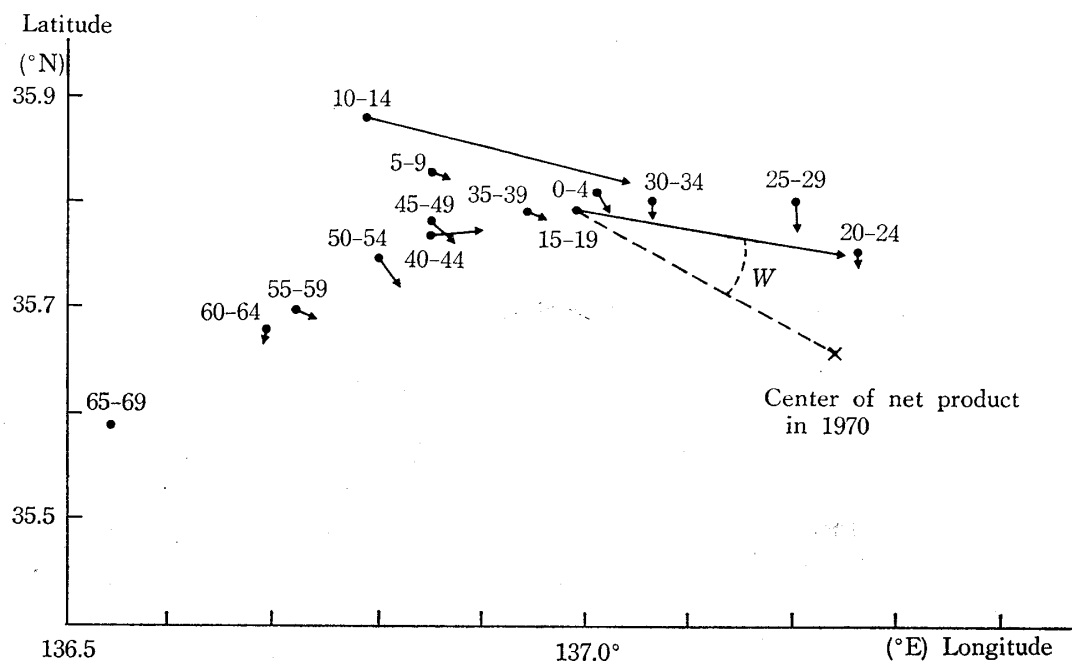
Figure 7 Movement of center of population for cohorts in Japan (1965-1970).
Note : The numbers in this figure show the years of age of cohorts. $x-x+4$
means the years of age of cohorts whose years of age are x, $x+1$, ..., $x+3$ and
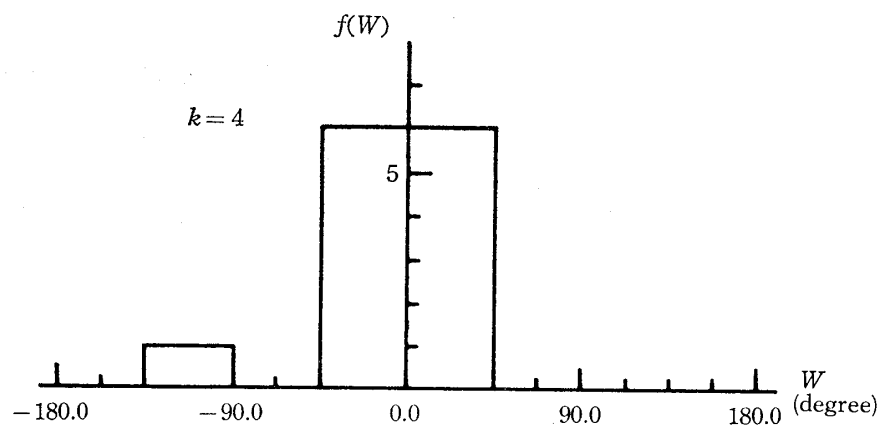$x+4$. The W in this figure is an example of measuring W.

Figure 8 Frequency distribution of direction of the movement of
population W.
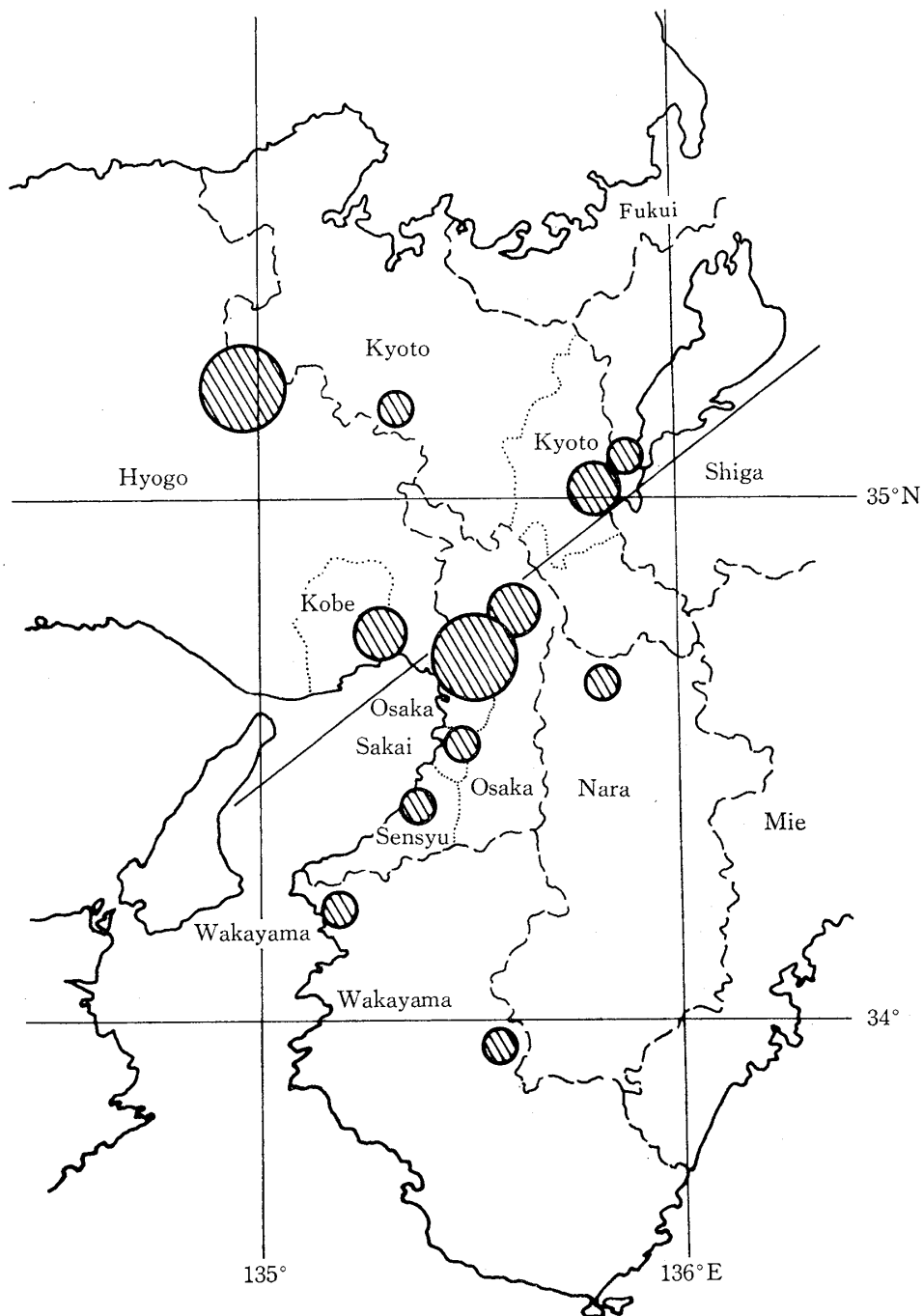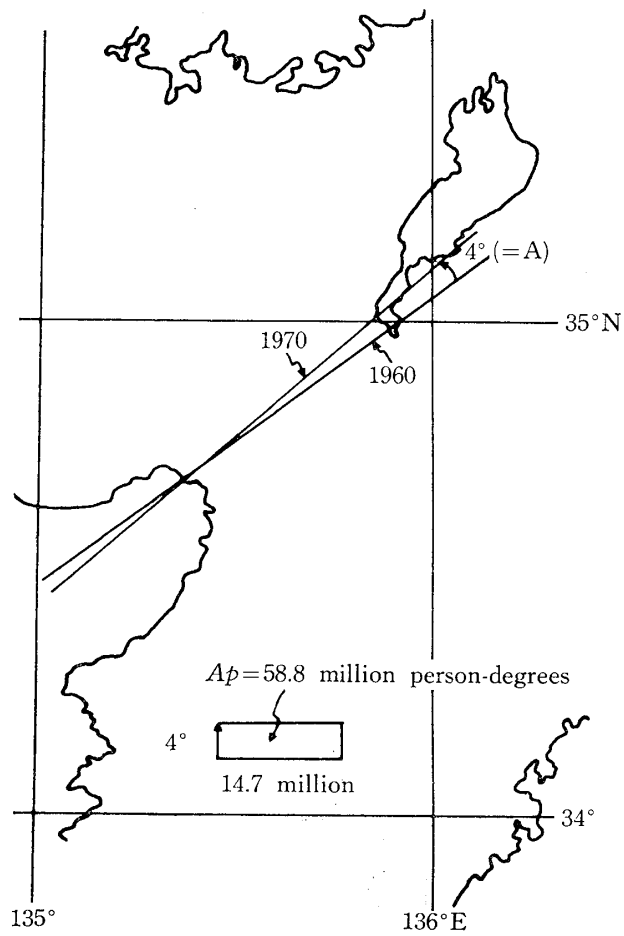Note : $f(W)$ is the frequency of W.

Figure 9 The axis of the population distribution in the regions surrounding Osaka (1960).

Note : In this figure, the area of a circle shows the population of a region. The circle is written at the location which is regarded as the central point of a region.

(1) Population

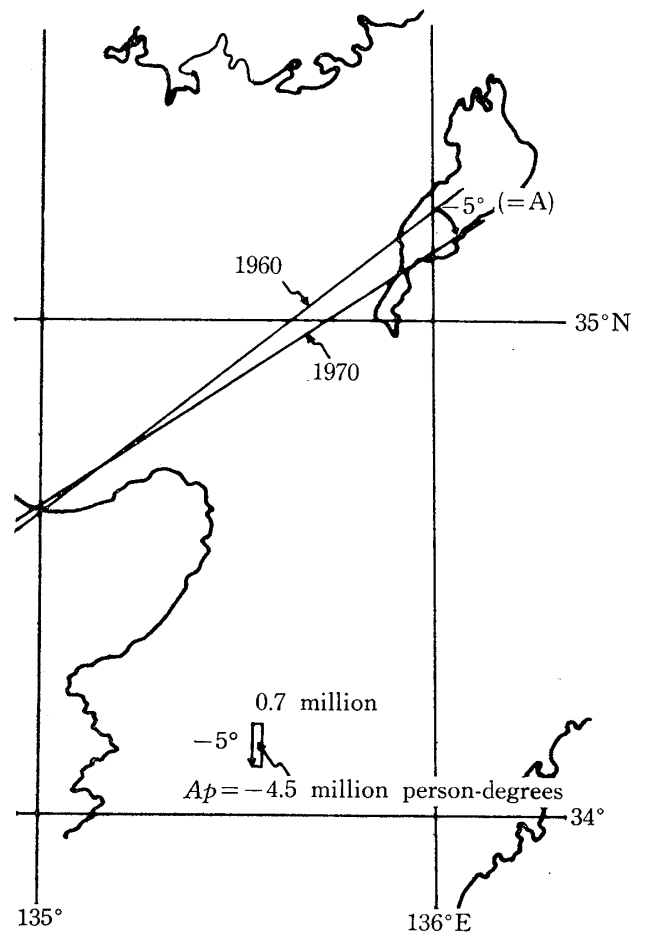(2) Population of the first sector of industry

Figure 10 The angle A for the population and the population of the first sector of industry in the regions surrounding Osaka.
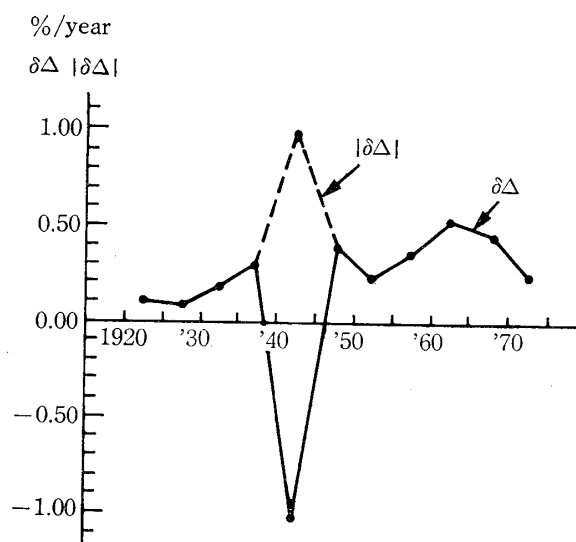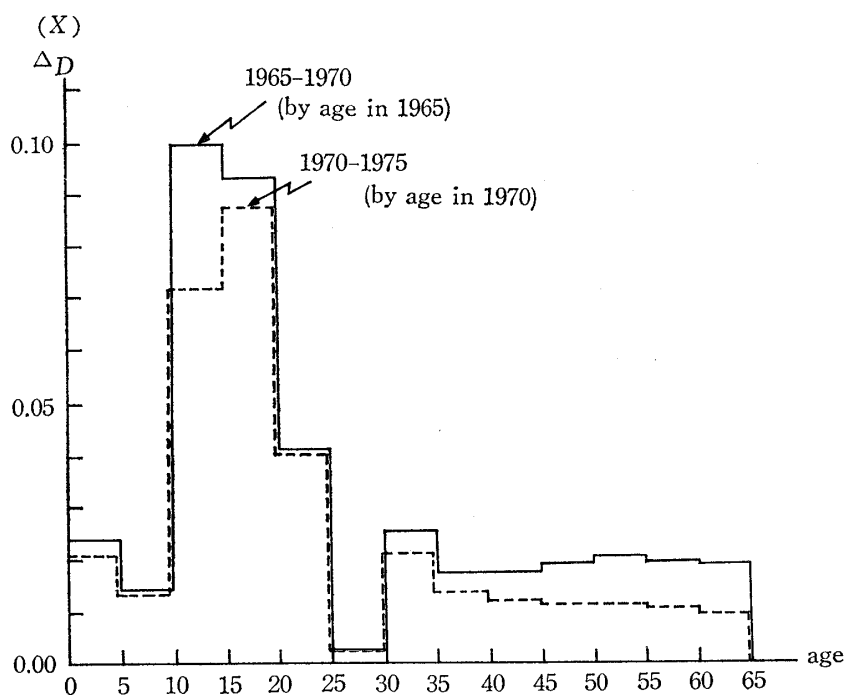

Figure 11 $\delta\Delta$ (and $|\delta\Delta|$) found in Japan.

Table 1 Hoover's concentration coefficient of population $\varDelta$ and $\delta\varDelta$ of Japan.

(1) $\varDelta$

Year	$\varDelta$
1920	29.68%
1925	30.21
1930	30.67
1935	31.65
1940	33.17
1945	28.14
1950	30.24
1955	31.43
1960	33.33
1965	36.06
1970	38.53
1975	39.87

(2) $\delta\varDelta$

Period	$\delta\varDelta$
1920–'25	0.106%
1925–'30	0.092
1930–'35	0.196
1935–'40	0.304
1940–'45	−1.006
1945–'50	0.420
1950–'55	0.238
1955–'60	0.380
1960–'65	0.546
1965–'70	0.494
1970–'75	0.268

Figure 12 The relationship between age and the value of $\varDelta^{(x)}{}_D$ for the periods from 1965 to 1970 and from 1970 to 1975.

Note: $\varDelta^{(x)}{}_D$ is the $\varDelta_D$ for the cohorts which have years of age from X to $X+4$, and the years of age expressed by those at the beginning of the period observed.

4. Some Problems related to Measuring Indicators

4-1 Sensitivity of the Indicators of Central Point of Population

The first center of population, namely, population center which can be used as a central point for measuring the indicators $\delta\varGamma$ and $\delta\varGamma_p$ is a meaningful point as a central point of population, because if the people observed gather at the point, when the people want to gather together at a point, the total distance of the movement becomes minimum, and this point is very significant

for determination of location of industrial activity.

But, unfortunately this central point has week sensitivity to the change of distribution of population. Therefore, when we use this point as an indicator of the characteristic of spatial distribution of population, this week sensitivity gives us a problem.

As a matter of fact, sometimes, the location of central point does not change, even if the state of spatial distribution of population changes. But, center of population, which can be also used as a central point for measuring the indicators $\delta\Gamma$ and $\delta\Gamma_P$, always move, when the state of spatial distribution of population changes.

Using a simple example, we can show the week sensitivity of population center.

First of all, we suppose that we have 5 persons who locate at 3 places P, Q and R whose co-ordinates are $(0, 0)$, $(3, 0)$, and $(7, 0)$ respectively, and we have 3 persons at P and 1 person at Q and R, where (x, y) is the symbol which expresses the locations x and y of a place on X-axis and Y-axis which are perpendicular to each other. In this case, the population center of the 5 persons is at P. On the other hand, the center of population is at the point C (namely at $(2, 0)$) as shown in Fig. 13 (A).

Next, we suppose that we have 10 persons who also locate at 3 places P, Q, and R, and we have 8 persons at P, and 1 person at Q and R. In this case, the population center of the 10 persons is also at P. But, the center of population is at C' (namely, at $(1, 0)$), as shown in Fig. 13 (B).
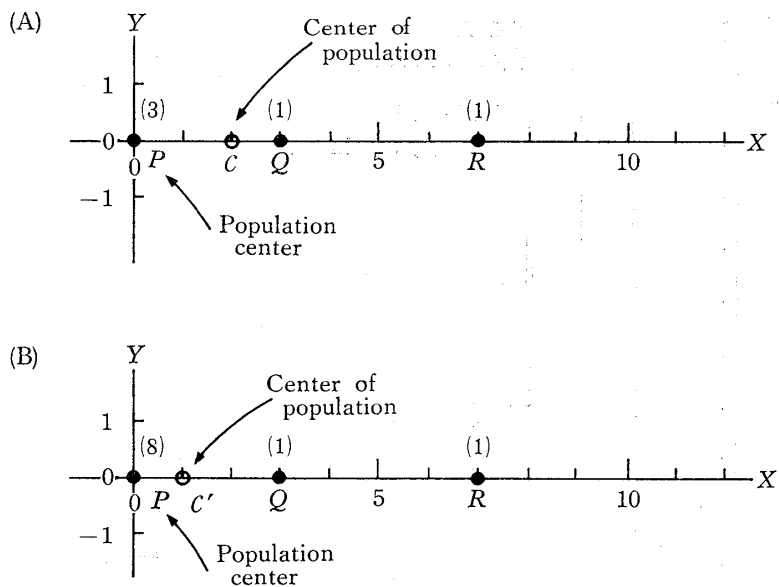

Figure 13 Population distribution, and its population center and center of population.
Note: The numbers in parentheses give the number of persons.

As shown by this example, we find sometimes the fact that even if the distribution of population changes, population center does not move. The reason why the population center does not move in this example is shown by the following explanation.

If the people gather at a point X which is at location $(x, 0)$, then the total distance of movement to gather at the point X, the total distance of movement D is expressed by

$$D = (3^{(persons)} \times x) + (1^{(person)} \times (3-x)) + (1^{(person)} \times (7-x))$$
$$= 10+x \tag{3.1}$$

for the first case, and

$$D = (8^{(persons)} \times x) + (1^{(person)} \times (3-x)) + (1^{(person)} \times (7-x))$$
$$= 10 + 6x \tag{3.2}$$

for the second case.

In both cases, when x is equal to 0, the D's of equations (3.1) and (3.2) are minimized.

In general, if we have n persons at the point P, then the total distance of movement for the persons to gather at the point X, D is expressed by

$$D = (n^{(persons)} \times x) + (1^{(person)} \times (3-x)) + (1^{(person)} \times (7-x))$$
$$= 10 + (n-2)x \tag{3.3}$$

Therefore, if n is larger than 2, $n-2$ becomes positive. Then, when n is larger than 2, the population center is always at the point P. On the contrary, the center of population is at location $(10/(n+2), 0)$, and it moves as the state of distribution of population changes.[20]

4–2 Determination of Axis of Spatial Distribution of Population

As already mentioned, the axis of spatial distribution of population can be determined by the method of principal component analysis. Essentially, the first axis of this analysis passes through the direction of maximum variance in the swarm of points which show the locations of persons.[21] Therefore, this axis can show the direction of maximum variance in the distribution of population. According to the method of the principal component analysis, the method for obtaining the axis for the variables X_1 and X_2 which are the latitude and the longitude of the location of a person, respectively, is stated as follows. When values observed for variable X_1 and X_2 are given by X, first of all, we must calculate the arithmetic means of these variable $\bar{X}$ and covariance matrix (namely, the matrix of variance and covariance of variable X_1 and X_2) S:

$$\bar{X} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \qquad S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

where

$$X = \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ \vdots & \vdots \\ x_{1n} & x_{2n} \end{bmatrix},$$

$$\bar{x}_p = \frac{1}{n} \sum_{i=1}^{n} x_{pi} \qquad ; (p = 1, 2)$$

$$s_{pq} = \frac{1}{n} \sum_{i=1}^{n} (x_{pi} - \bar{x}_p)(x_{qi} - \bar{x}_q) ; (p = 1, 2 ; q = 1, 2)$$

and x_{pi} ($p=1, 2$; $i=1, 2, ..., n$) is the ith value observed of the variable X_p ($p=1, 2$).

Secondly, we calculate the characteristic vectors a_1 and a_2 of the equation:

$$(S - \lambda I)a = 0 \tag{3.4}$$

In this case,

$$a_1 = \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}, \qquad a_2 = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}$$

and a_1 is the characteristic vector for the largest characteristic root λ_1, and a_2 is the characteristic vector for the smaller characteristic root λ_2.

Incidentally, these characteristic vectors are the coefficients of X_1 and X_2 for expressing the first axis Y_1 and the second axis Y_2. The axes Y_1 and Y_2 are expressed by

$$Y_1 = a_{11}X_1 + a_{12}X_2 \tag{3.5.1}$$

$$Y_2 = a_{21}X_1 + a_{22}X_2 \tag{3.5.2}$$

Lastly, if we calculate the values a_1, b_1, a_2, and b_2 by the following equations, we can obtain

the equations which express the axes Y_1 and Y_2 by the variables X_1 and X_2.

$$a_1 = -\frac{a_{12}}{a_{11}} \qquad (3.6.1)$$

$$a_2 = -\frac{a_{22}}{a_{21}} \qquad (3.6.2)$$

$$b_1 = \bar{x}_1 + \frac{a_{12}}{a_{11}}\bar{x}_2 \qquad (3.6.3)$$

$$b_2 = \bar{x}_1 + \frac{a_{22}}{a_{21}}\bar{x}_2 \qquad (3.6.4)$$

Then, the first axis is expressed by

$$x_1 = a_1 + b_1 x_2 \qquad (3.7.1)$$

and the second axis is expressed by

$$x_1 = a_2 + b_2 x_2 \qquad (3.7.2)$$

But, the data of population distribution is usually given in the form of population in the regions observed. Therefore, the values observed are given in the following form:

$$X = \begin{bmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \\ \vdots & \vdots \\ x_{1m} & x_{2m} \end{bmatrix}, \qquad P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_m \end{bmatrix}$$

where P_j $(j=1, 2, ..., m)$ is the population in the jth region whose location is shown by latitude x_{1j} and longitude x_{2j} (namely, location (x_{2j}, x_{1j})).

In this case, the method for obtaining the axes of the distribution of population is stated as follows.

First of all, we calculate the elements of the vector $\bar{X}$ and the matrix S by

$$\bar{x}_p = \frac{1}{\sum\limits_{j=1}^{m} P_j} \sum_{i=1}^{m} P_j x_{pj}$$

$$s_{pq} = \frac{1}{\sum\limits_{j=1}^{m} P_j} \sum_{i=1}^{m} P_j (x_{pj} - \bar{x}_p)(x_{qi} - \bar{x}_q)$$

After we obtain these values, if we calculate the a_1 and a_2 by equation (3. 4), we can obtain the axes by the same calculation which are expressed by equations (3. 6. 1), (3. 6. 2), (3. 6. 3) and (3. 6. 4) written above.

When we calculate the axes with the data of population of the regions observed whose location are expressed by latitude and longitude, even if we can obtain the two axes by the method shown above, these axes are not perpendicular to each other on a real map. As far as, we use the longitude and the latitude when we express the location of regional population, these axes are not perpendicular to each other, since the length of one degree of longitude is not necessary equal to that of latitude. But, each axis can be used as the indicator for expressing the state of distribution of population.[22]

5. Conclusion

In this paper, first of all, I wrote down many kinds of indicators for representing the profile of spatial distribution of population, especially indicators for representing the characteristics of the change of the distribution of population, and classified these indicators into 3 kinds (Fig. 14).

Secondly, I showed the results obtained by measuring the change of population distribution by using the indicators mentioned here.

And lastly, I discussed interesting problems which we would find when we used the indicators proposed here.

When I tried to apply the indicator for measuring the characteristics of the change of distribution of population, I was able to find that the indicators were fairly good indicators to grasp the characteristics.
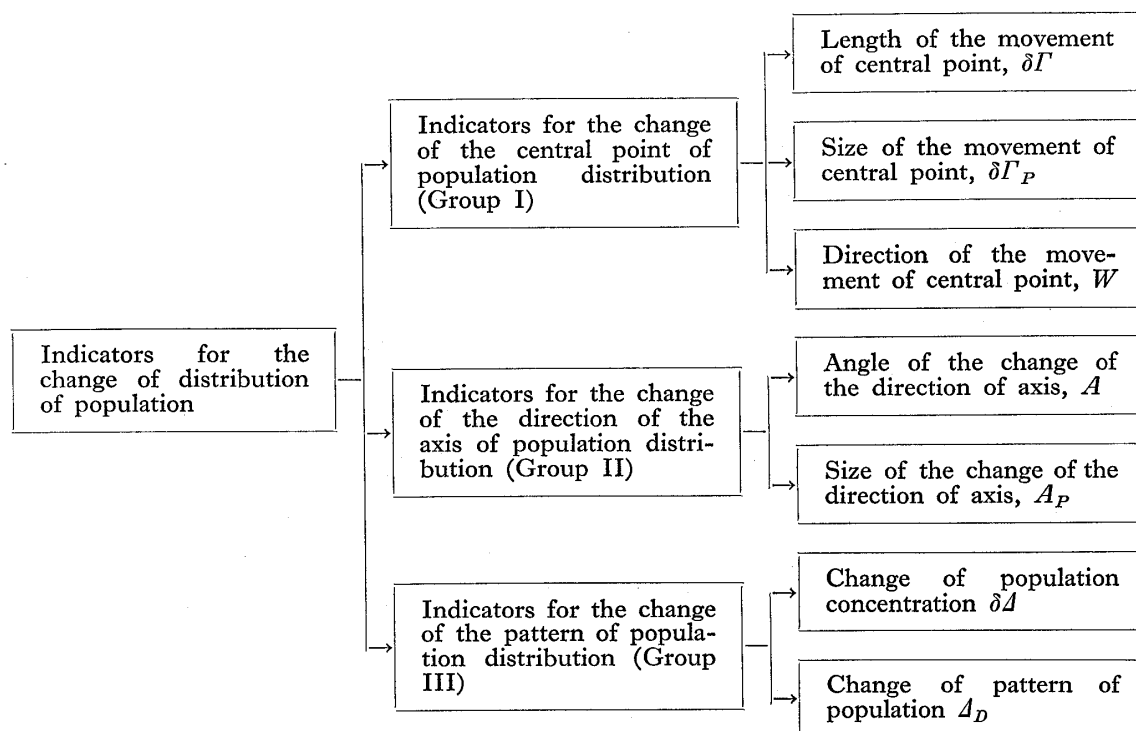

Figure 14　Systematic representation of the proposed indicators.

Notes

* Main part of this paper was presented to the 24th International Geographical Congress held at Tokyo from September the 1st to the 5th, 1980 (Suzuki, Keisuke: Indicators for measuring the characteristics of the geographical distribution of population, 24th *International Geographical Congress, Main Session, Abstracts*, Vol. 2, 1980, pp. 230-231.).

Some of the calculations of the analysis performed in this paper were done at the Information Science Research Center, Aoyama Gakuin University, FUJIMIC (Fuji Management Information Center) and FUYO (Fuyo Data Processing and Systems Development, Ltd.). I am indebted to Mr. Masanori Miyata and Mr. Shigeru Hongo, assistants of Aoyama Gakuin University, Mr. Hiroshi Wakui and Mr. Taizo Hamana, Shakai Shisutemu Kenkyu-sho (Social System Research Institute), and Mr. Masaharu Tsuchiya, Mr. Koji Kaba and Mr. Fumiyuki Fujiwara, FUYO for their technological assistance on computation by electronic computers.

1) Neft has already written an excellent book on the indicators of the distribution of population. In his book, he discussed his " areal moment."

Neft, David S.: " *Statistical Analysis for Areal Distribution*," Philadelphia, Regional Science Research Institute, 1966.

Suzuki, Keisuke: *Kukan Jinkogaku* (*Space Demography*), Tokyo, Taimei-do, 1980 (In Japanese with English preface), pp. 74-77.

2) Tachi, Minoru: *Keishiki Jinkogaku* (*Formal Demography*) Tokyo, Kokon Shoin, 1960 (In Japanese with English Preface), pp. 444-445.

Kroeber, A. L.: " Native American Population," *American Anthropologist*, Vol. 36, No. 1, 1934.

Kroeber, A. L.: *Cultural and Natural Areas of Native North America*, University of California Press, 1947, pp. 166–172.

Stanner, W. E. H.: *The South Seas in Transition, a Study of Post-War Rehabilitation and Reconstruction in Three British Pacific Dependencies*, Sydny, 1953.

3) Tachi, Minoru: *op. cit.* p. 430.

Mayr, Georg von: *Statistik und Geselschaftslehre*, Erste Band, *Theorestische Statistik* (2 Aufl.), Tübingen, 1941, S. 161.

Flaskämper, Paul: *Bevölkerungsstatistik*, Hamburg, Ferix Meiner, 1962, S. 102.

4) Tachi, Minoru: *op. cit.* pp. 451–454.

Flaskämper, Paul: *Bevölkerungsstatistik*, *op. cit.* S. 102 f.

5) Duncan ,Otis Dudley, Ray P. Cuzzort and Beuerly Duncan: *Statistical Geography*, Illinois, Free Press of Glencoe, 1961, p. 83.

6) Greenwald, William I.: *Statistics for Economics*, Columbus, Ohio, Charles E. Merril, 1963, pp. 24–26.

Yasuda, Saburo: *Shakai Tokeigaku (Social Statistics)*, Tokyo, Maruzen, 1969, pp. 276–284.

7) Kunimatsu, Hisaya, Masuo Ando, Hisao Nishioka, Keisuke Suzuki, Takashi Okuno: *Keizai Chirigaku (Economic Geography)*, Tokyo, Meigen Shobo, 1971, pp. 273–274.

8) Neft, David S.: *op. cit.*

Suzuki, Keisuke: *op. cit.*

9) If all the persons moves as long as $\delta\Gamma$ in a given direction, then the central point also moves as long as $\delta\Gamma$ to the given direction and the total length of the movement of the persons is exactly equal to $P(\delta\Gamma)$ which is the value of the indicator $\delta\Gamma_P$, where P is the number of the persons observed. Consequently, the size of the movement of central point $\delta\Gamma_P$ is the total length of the movement of people under the supposition that all the persons moves as long as $\delta\Gamma$ in a give direction.

10) Kuhn, H. W. and R. E. Kuenne: "An efficient algorithm for the numerical solution of the generalized Weber problem in spatial economics," *Journal of Regional Science*, Vol. 4, No. 2, 1962, pp. 21–33.

11) Suzuki, Keisuke: *op. cit.*, pp. 47–55, pp. 344–349.

Suzuki, Keisuke: A historical review of the study of the " central point of population," *The Journal of Ryūtsū Keizai University*, Vol. 12, No. 2, 1977, pp. 1–37 (In Japanese with English synopsis).

Suzuki, Keisuke: " Statistical indicators of the movement of the position of spatial distribution of population: a statistical analysis of regional data of population," *The Journal of Ryūtsū Keizai University*, Vol. 14, No. 1, 1979, pp. 34–47.

12) If all the persons move as widely as A°, then the axis moves also as widely as A° and the total angle of movement of the persons is exactly equal to P A°, which is the value of the indicator A_P. Therefore, the change of the direction of the axis A_P is the total angle of the movement of people under the supposition that all the persons move as widely as A°.

13) Suzuki, Keisuke: *op. cit.* (1979, Vol. 14, No. 1).

14) The center of population for cohorts are calculated from prefectural data of Japan.

Suzuki, Keisuke: " The characteristics of the changes of the distribution of population of cohorts in Japan," *The Journal of Ryūtsū Keizai University*, Vol. 14, No. 2, 1979.

15) Suzuki, Keisuke: *op. cit.*, (1979, Vol. 14, No. 2).

16) Suk-han Shin also tried to use axis to represent the distribution of population.

Suk-han Shin: "Amenity resources and residential blight: Defining and identifying urban residential quality in space" Presented at the 6th Annual Pacific Regional Science Conference, Seul, Korea, August 14, 1979. (Mimeograph)

17) Exactly saying, the population and the population of the first sector of industry in the regions observed in 1965 were 14,687,843 and 909,826, respectively.

Therefore, A_P for these populations were as follows:

$$A_P \text{ (for population)} = 14{,}687{,}843 \times 4°$$
$$= 58{,}751{,}372 \text{ (person-degrees)}$$

$$A_P \text{ (for the population of the first sector of industry)} = 909{,}826 \times (-5°)$$
$$= -4{,}549{,}130 \text{ (person-degrees)}$$

18) Suzuki, Keisuke: *op. cit.* (1979, Vol. 14, No. 1).

19) Suzuki, Keisuke: *op. cit.* (1979, Vol. 14, No. 2.).

20)　It can be said that the week sensitivity of population center suggests that if a great town once appears in the regions observed, the location of the great town becomes a stable central point and it can be stationarily the central point in the region though the population distribution in the regions surrounding the great town changes a little.

21)　Morrison, Donald F.: *Multivariate Statistical Methods*, New York, McGraw-Hill, 1967, pp. 230–234.

22)　Even if we intend to get the axes which are perpendicular to each other on a real map, exactly saying, it is very difficult to obtain such axes, since the surface of the earth is not plane, but spherical.

For example, the first and second axes for the distribution of population for 1960 were as follows:

$$\begin{cases} x_1 = 0.25 + 0.84 x_2 & \text{(The first axis)} \\ x_1 = 10.10 - 0.97 x_2 & \text{(The second axis)} \end{cases}$$

But, the axes obtained by these equations are not perpendicular to each other on a real map, because the length of the 1 degree of longitude is not equal to that of latitude.

要　　約

鈴木啓祐:「地域的人口分布の変化の特徴を示すための定量的指標について」『流通経済大学論集』第16巻第1号, 1981年 1–16 ページ。

この論文は, 1980年にわが国においておこなわれた The 24th International Geographical Congress に発表した報告を基礎として書かれたものである。ここでは, 特に, 地域的人口分布の時間的変化の特徴を定量的にとらえるために筆者が提案するいくつかの指標(人口分布変化指標)——このうちのいくつかは, すでに筆者が他の場所で提案したものである——を体系的に叙述し, その適用例を列挙した。

ここに提案する指標は, まず, 大きく,

（1）　第1指標群: 人口分布の中心の変化に関する指標

（2）　第2指標群: 人口分布の軸の変化に関する指標

（3）　第3指標群: 人口の分布様式の変化に関する指標

の3種類に大別される。

第1指標群に属する指標としては,

（i）　人口中心(すなわち, 人口分布の中心的位置)の移動距離 $\delta\Gamma$

（ii）　人口中心の移動規模 $\delta\Gamma_P$

（iii）　人口中心の移動方向 W

が, 第2指標群に属する指標としては,

（i）　人口軸(すなわち, 人口分布の軸の位置)の方向変化量 A

（ii）　人口軸の方向変化規模 A_P

が, そして, また, 第3指標群に属する指標としては,

（i）　人口集中性の変化量 $\delta\Delta$

（ii）　人口分布様式の変化量 Δ_D

が, それぞれ挙げられている。

これらの人口分布変化指標の適用例としては, すべてわが国の人口に対する適用例を挙げた。その適用例においては, わが国の人口移動は, 年齢別に見ても, わが国の純生産の中心(地域別純生産の重心)に向って生じていることや, わが国の人口分布が第2次世界大戦の期間において, 急激に地域的集中性を弱めたりすることが, 人口分布変化指標を用いて明確にとらえられることを示した。

また, 人口分布変化指標に関係する問題点も指摘した。

そのうちの第1のものは, 人口中心の移動距離 $\delta\Gamma$ や移動規模 $\delta\Gamma_P$ の測定の際に, 人口中心の指標として用いられ得る人口中心点 (population center) は, やはり, $\delta\Gamma$ や $\delta\Gamma_P$ の測定の際に人口中心の指標として用いられ得る人口重心 (center of population) よりも, 人口分布の変化に対して敏感性が低いという点である。実際, 人口中心点を $\delta\Gamma$ や $\delta\Gamma_P$ のための人口中心の指標として用いると, 人口分布が変化しても, その変化が $\delta\Gamma$ や $\delta\Gamma_P$ の上に定量的に反映されて来ない場合があるのである。

ところで, このような人口中心点のもつ性質が, ある地域の中心的都市が安定的に存在し得る理由を示しているように思えた。いま, もし, ある地域の中心的都市が, その地域の人口中心点に出現するならば, その地域の人口がある条件——ごく, 簡単にいえば, その中心的都市以外に分布する全人口がその中心的都市の人口よりも比較的小さいという条件——の下に変化しても, その地域の人口中心点は変化しない。したがって, その地域の人口中心点にあった中心的都市は, 人口分布が変化した後においても, 人口中心点の位置にあり, 中心的都市として存続し得るのである。中心的都市が永続的に存在し得る理由が, この中心的都市

が比較的安定的な人口中心点，あるいは，これに近い位置に出現する点にある，いいかえれば，中心的都市の適地が人口中心点，あるいは，これに近い地点であり，人口中心点は安定的であることからその適地も安定的である点にあることを示唆しているように思えた。

　第2のものは，人口分布の軸の決定に関する問題である。人口分布の軸は，種々の方法で決定される可能性があるが，ここでは，それを主成因分析 (principal component analysis) の第1軸——地域的人口の位置を変量とみなすことによって得られる——によって示すことにした。しかしながら，この軸を決定するとき，

人口の位置を緯度と経度を用いて表現するかぎり，主成因分析による人口分布の第1軸と第2軸は，実際の地図上では直交しない。その理由は緯度の1度の長さと経度のそれとは一般に互いに異っているからである。しかし，地球の表面が球面であるので，厳密に言って，実際の地図の上で直交する主成因分析の2軸を得ることは困難であるといえよう。したがって，ここでは，一般に主成因分析の第1軸のみによって人口分布の軸を表現することにした。

　各種の人口分布変化指標を実際の人口分布の解析に適用したところ，これらの指標がかなり有効な指標であることが知られた。