The Theoretical Explanation of the Mechanism of Existence of Overlapping Market Area for Two Retailers

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1. Introduction

When we discuss "market area" or "boundary of market area", we cannot forget three students who were great pioneers of the study of this problem. The students are William Launhardt, Harold Hotelling and Tord Palander.

Launhardt who showed first examination of industrial location[1] discussed a market area for a producer located at a given point in a market[2].

Hotelling studied the change of boundary of market area of two retailers which occurs by the change of the locations of the two retailers R_A and R_B who endeavor to get their market as widely as possible. From his study of the "spatial competition" which is found between the two retailers, he concluded that the two retailers would stand back to back in the center of the market[3].

And Palander examined the effect of the price at the location of producer of commodity and cost of transportion per km (or mile) to market areas of two sellers[4].

According to Hotelling and Palander, the boundary of market areas for sellers R_A and R_B is given at the location where the condition expressed by the following equation is satisfied.

$$p_A+f_A\overline{AC}=p_B+f_B\overline{BC} \qquad (1.1)$$

or

$$\begin{cases} P_A=P_B & (1.2.1) \\ P_A=p_A+f_A\overline{AC} & (1.2.2) \\ P_B=p_B+f_B\overline{BC} & (1.2.3) \end{cases}$$

where P_A and P_B are the prices of the commodity solled by the sellers R_A and R_B at a point in market C, respectively; p_A and p_B the prices of the commodity at the locations of the sellers R_A and R_B, A and B; f_A and f_B the cost of transportation per km, namely, freight rates, for obtaining the commodities sold by the sellers R_A and R_B; and $\overline{AC}$ and $\overline{BC}$ the distances from the location of seller R_A, A to a point in market C and from the location of seller R_B, B to the point C.

If a market in which the markets for the sellers R_A and R_B appear is a straight line drawn from point L to point M, the boundary of market areas can be shown by the point C^* in Figure 1, as already shown by Palander and Hotelling (Figure 1). If we denote the values of distances $\overline{AC}$ and $\overline{BC}$ which satisfy the condition given by equation (1.1) or given by equations (1.2.1), (1.2.2) and (1.2.3), by $\overline{AC^*}$ and $\overline{BC^*}$, andt he point C at which the

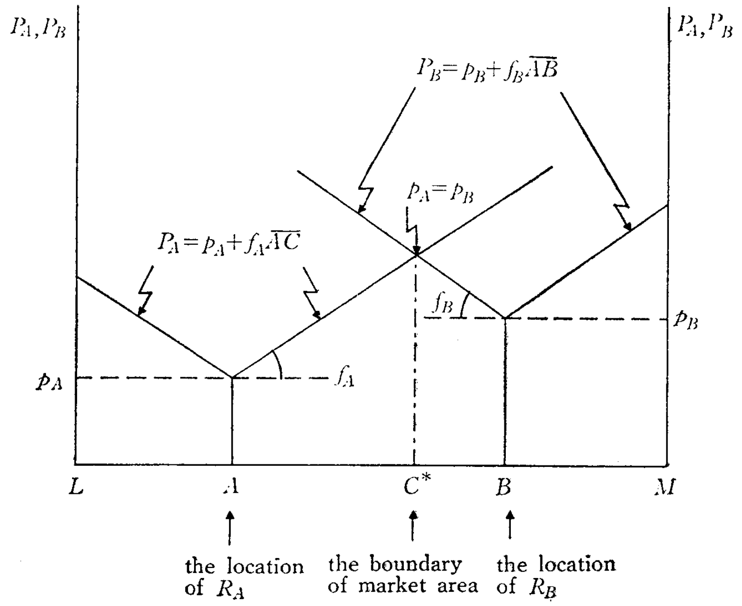


Figure 1 The boundary of market areas of sellers R_A and R_B,

distances AC and BC become distances $\overline{AC^*}$ and $\overline{BC^*}$, by C^*, then C^* is a point which shows the boundary between the markets for sellers R_A and R_B.

In this paper, I examined a modification of the model used by Palander and Hotelling and tried to find theoretically overlapping market area[5].

2. Spatial Cost Line for Each of n Pieces of a Commodity

If the price of a commodity G sold by a retailer R_A at the location of this retailer A is p_A and freight rate is f_A, then the total cost for a consumer at location X who purchases the commodity, P_A is expressed by

$$P_A = p_A + f_A d \tag{2.1}$$

where d is the distance between the location A and the location X.

Actually, when a consumer at location X who wants to purchase one piece of the commodity G, he must expend his money as much as P_A.

But, if the consumer wants to purchase two pieces of the commodity G, and if we additionally suppose that a half of the total cost of transportation is allocated equally to each piece of the commodity, then the total cost for purchasing one piece of the commodity, namely, the price of the commodity for the consumer at location X, $P_A(2)$ must be expressed by

$$P_A(2) = p_A + \frac{1}{2} f_A d \tag{2.2}$$

In general, if the consumer at location X wants to purchase n pieces of the commodity G from the retailer R_A, and if we suppose that one nth of the total cost of transportation is again allocated equally to each pieace of the commodity, then the price of the commodity at location X, $P_A(n)$ can be expressed by

$$P_A(n)=p_A+\frac{1}{n}f_Ad\,;\ n=1, 2, \cdots,\ n \qquad (2.3)$$

When n is equal to 1, equation (2.3) becomes equation (2.1).

In Figure 2, we can find the lines showing the relationship between the distance d and the price $P_A(n)$ (n=1, 2, N).

In this paper, each of the lines is called "*spatial cost line for each of n pieces of commodity*".

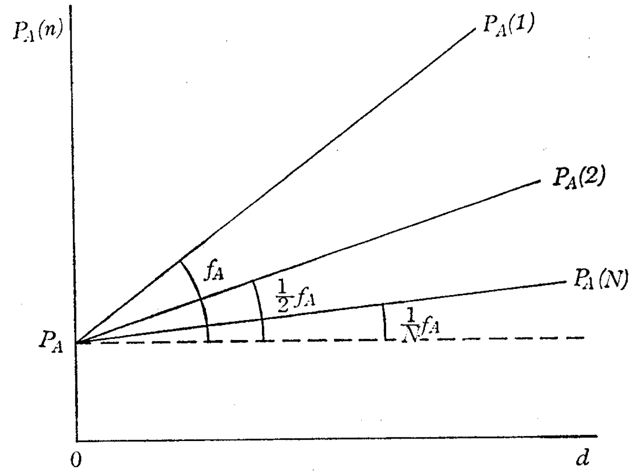


Figure 2 Spatial cost lines for each of n pieces of commodity (n=1, 2, N).

3. Spatial Cost Line for a Commodity among m Kinds of Commodities

In this section, the total cost for for purchasing a commodity G_1, for a consumer at location X who wants to purchase simultaneously m kinds of commodities $G_1, G_2, \cdots, G_i, \cdots, G_m$ from the retailer R_A (or retailers $R_{A1}, R_{A2}, \cdots, R_{Am}$ who sell the commodities $G_1, G_2, \cdots, G_i, \cdots, G_m$, respectively) at location A is examined.

Here, if it is also supposed that when the consumer purchases m kinds of commodities at location A, total cost of transportation is allocated equally to a commodity G_1, then the price of the commodity G_1 for the consumer at location X, P_A $(G_1/G_1, G_2, \cdots, G_m)$ must be expressed by

$$P_A(G_1/G_1, G_2, \cdots, G_m)=p_A(G_1)+\frac{1}{m}f_Ad \qquad (3.1)$$

where $p_A(G_1)$ is the price of the commodity G_1 at the location A.

When m is equal to 1, the form of equation (3.1) becomes that of equation (2.1).

We can also draw a line which depicts the relation between the distance d and the price $P_A(G_1/G_1, G_2, \cdots, G_m)$ $(m=1, 2, M)$ as shown in Figure 3. Each of the lines is called, here "*spatial cost line for a commodity among m kinds of commodities*".

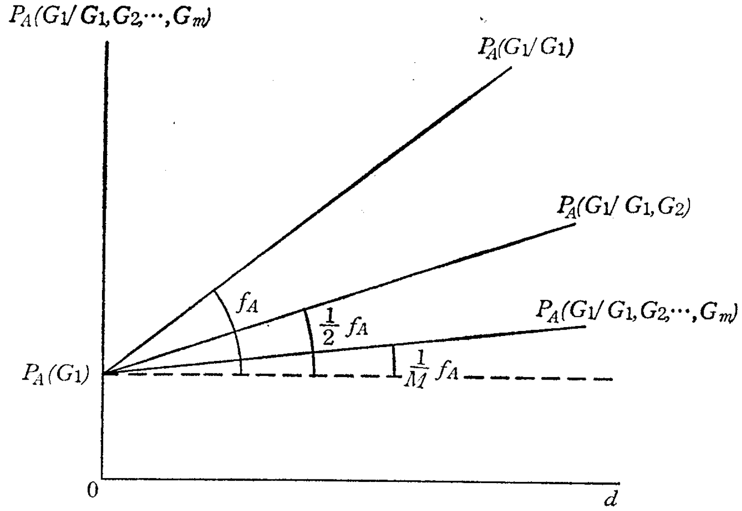


Figure 3 Spatial cost lines for a commodity among m kinds of commodities $(m=1, 2, M)$.

The form of the spatial cost line for a commodity among n kinds of commodities is expressed by the same form as the spatial cost line for each of n pieces of a commodity.

4. Market Areas for Retailers R_A and R_B

When we theoretically determine the market area for two retailers R_A and R_B by using the spatial cost line for a commodity among m kinds of commodities, we can find overlapping market area for the two retailers R_A and R_B at location A and B.

For simplicity, here, I try to examine the boundary of market areas under the supposition that (1) the retailer R_A at location A sells two kinds of commodities G_1 and G_2, and the retailer R_B at location B sells only one kind of commodity G_1 and (2) a person belonging to a group of consumers, a consumer of Type I, at location X always wants to purchase only a piece of commodity G_1 and a person belonging to another group of consumers, a consumer of Type II, at location X wants to purchase simultaneously two pieces of commodities which consists one piece of commodity G_1 and one piece of commodity G_2.

When a person is a consumer of Type I and purchases only a piece of commodity G_1 from the retailer R_A and R_B, the spatial cost line for one piece of the commodity G_1 is, as already shown by equations (1.2.2) and (1.2.3) in section 1, expressed by the following equation :

$$P_A(G_1)=p_A(G_1)+f_A d_A \qquad (4.1.1)$$
$$P_B(G_1)=p_B(G_1)+f_B d_B \qquad (4.1.2)$$

where $P_S(G_1)$ $(S=A, B)$ is the price of the commodity G_1 sold by a retailer R_S $(S=A, B)$

at location S for the consumer at location X; $p_S(G_1)$ the price of the commodity G_1 sold by the retailer R_S at the location of the retailer R_S, S; f_S the freight rate for obtaining the commodity sold by the retailer R_S; and d_S the distance from the location of the consumer X to the location of the retailer R_S.

At the boundary of the market areas of the commodity G_1 for retailers R_A and R_B, the condition which is expressed by the following equations must be satisfied.

$$P_A(G_1)=P_B(G_1) \qquad (4.2.1)$$

or

$$p_A(G_1)+f_Ad_A=p_B(G_1)+f_Bd_B \qquad (4.2.2)$$

And, if the market area observed is a straight line drawn from point L to point M, as shown in Figure 4, the boundary can be shown by a point C^*.

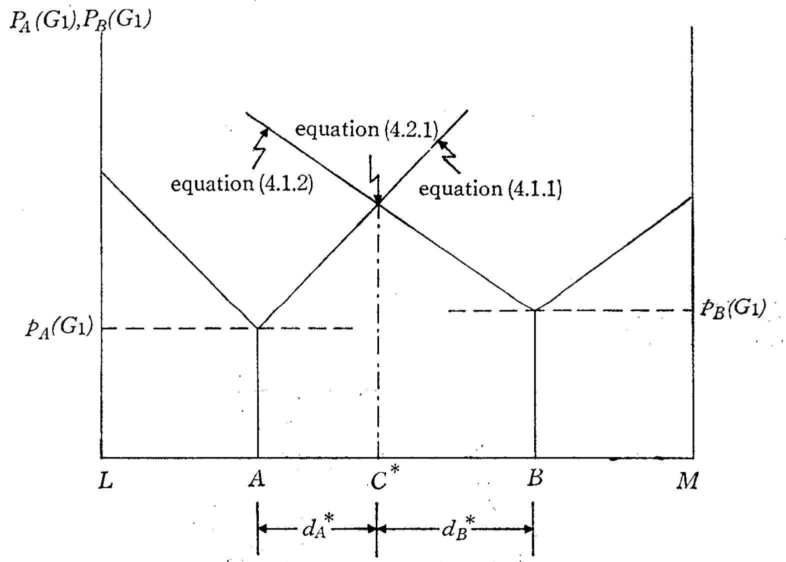


Figure 4 The boundary of market areas for the retailers R_A and R_B in the market occupied by the consumers of Type I.

Note : d_A* and d_B* are the distances from location A to location C* and from location B to location C. Equations (4.1.1), (4.1.2) and (4.2.1) in this graph are equations (4.1.1), (4.1.2) and (4.2.1) in this paper, respectively.

On the other hand, when a person is a consumer of Type II and purchases one piece of each of the commodities G_1 and G_2, provided that he can find the both commodities, he will purchase one piece of each of commodities G_1 and G_2 from retailer R_A at location A, but he purchases only one piece of the commodity G_1 from retailer R_B at location B.

Therefore, we can draw a spatial cost line for a commodity among 2 kinds of commodities, namely, the spatial cost line for the commodity G_1 (exactly saying, for one piece of the commodity G_1) among two kinds of commodities, G_1 and G_2 from the retailer R_A for the consumer of Type II. This cost line is expressed by

$$P_A(G_1/G_1,\ G_2)=p_A(G_1)+\frac{1}{2}f_Ad_A \qquad (4.3.1)$$

But, we cannot draw a spatial cost line for a commodity among two kinds of commodities for the consumer of Type II from the retailer R_B. We can only draw the spatial cost line for one piece of commodity G_1 from the retailer for the consumer of Type II from the retailer R_B, because he cannot find the both commodities at location B where the retailer R_B stands. The spatial cost line for a one piece of commodity G_1 is expressed by

$$P_B(G_1)=p_B(G_1)+f_Bd_B \qquad (4.3.2)$$

Therefore, at the boundary of the market areas of the commodity G_1 for retailers R_A and R_B, the condition which is expressed by the following equations must be satisfied.

$$P_A(G_1/G_1,\ G_2)=P_B(G_1) \qquad (4.4.1)$$

or

$$p_A(G_1)+\frac{1}{2}f_Ad_A=p_B(G_1)+f_Bd_B \qquad (4.4.2)$$

Here, we can obtain another boundary of market areas for the retailers R_A and R_B

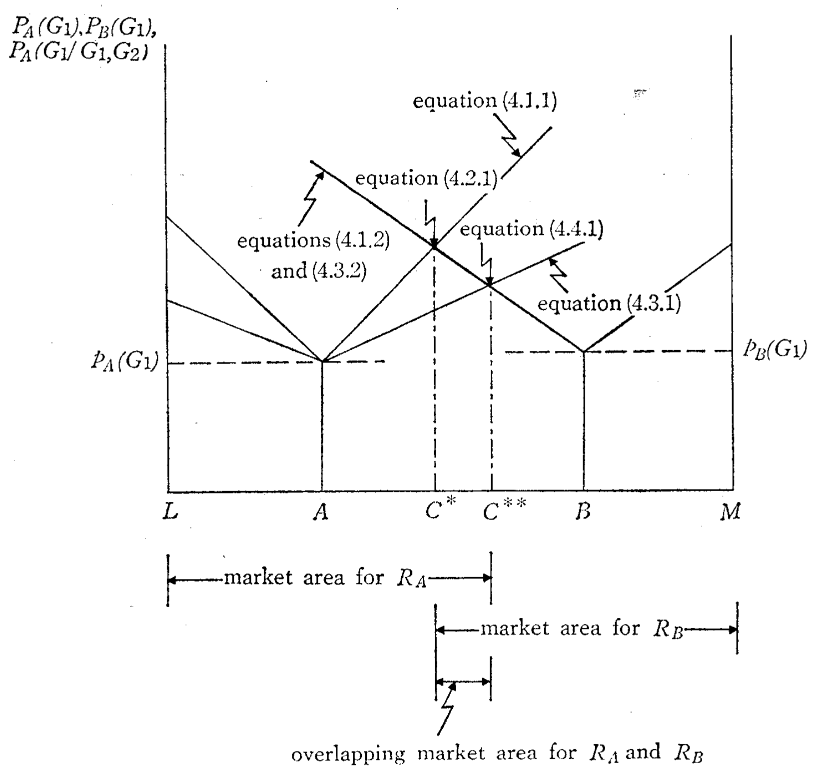


Figure 5 The boundaries of market areas for the retailers R_A and R_B in the market occupied by the consumers of Type I and Type II.

under the condition expressed by equations (4. 4. 1) or (4. 4. 2) which is shown by point C^{**} in Figure 5.

In Figure 5, we can find two boundaries of market areas, the boundary obtained under the condition expressed by equation (4. 2. 1) (or equation (4. 2. 2)) which is determined by consumers of Type I and that obtained under the condition expressed by equation (4. 4. 1) (or equation (4. 4. 2)) which is determined by consumers of Type II. Therefore, the part of market from C^* to C^{**} in Figure 5 is the market area for retailer R_B when the retailer R_B sells his commodity G_1 to a consumer of Type I and simultaneously, it is the market area for retailer R_A when the retailer R_A sells his commodity G_1 to a consumer of Type II. The part of market from C^* and C^{**} in Figure 5 is an overlapping area for the retailers R_A and R_B.

Consequently, from a theoretical consideration we can say that there is the possibility of appearance of overlapping market area for retailers R_A and R_B.

5. Examples Obtained from Survey

According to the result of a recent survey which was done by *the Kichijōji Station Center*[6)], the consumers who came to Kichijōji which have a considerable great shopping center to buy commodities were found in a very wide area, as shown in Figure 6.

This result suggests the existence of overlapping area of retailers. Therefore, we can say that this is regarded as an "indirect evidence" for the existence of overlapping market area of retailers.

I can show another example which is regarded as a "direct evidence" for the existence of the overlapping market area. In the areas in the vicinity of Kichijōji Station and Musashisakai Station, we can find many shop-lined streets. Figure 7 (a) and (b) show the market areas for some group of retailers of the shop-lined streets in the areas in the vicinity of Kichijōji Station and Musashisakai Station which were obtained from the survey performed by the Musashino Chamber of Commerce and Industry[7)].

According to these figures, we can find directly the overlapping market areas for some groups of retailers. For example, the areas O_1 and O_2 in Figure 7 (a) and the area O in Figure 7 (b) are the overlapping market area of the market areas for the shop-lined streets $S(1)$, $S(2)$ and $S(4)$ and for the shop-lined streets $S(7)$, $S(8)$, $S(9)$, $S(10)$ and $S(11)$; that of the market areas for the shop-lined streets $S(7)$, $S(8)$, $S(9)$, $S(10)$ and $S(11)$ and for the shop-lined streets $S(12)$ and $S(26)$; and that of the market areas for shop-lined streets $S(51)$, $S(52)$ and $S(53)$ and for the shop-lined streets $S(54)$, $S(55)$, $S(56)$ and $S(57)$, respectively.

From this result of the survey, we can find actually the size and configuration of the overlapping market area.

It is very interesting to find that some parts of boundary of a market area are in the overlapping market area and other parts of it are not in the overlapping market area.

This show that the overlapping areas found in these maps do not necessary appear by physical insufficiency of the number of the kinds of commodities. If the reason why the

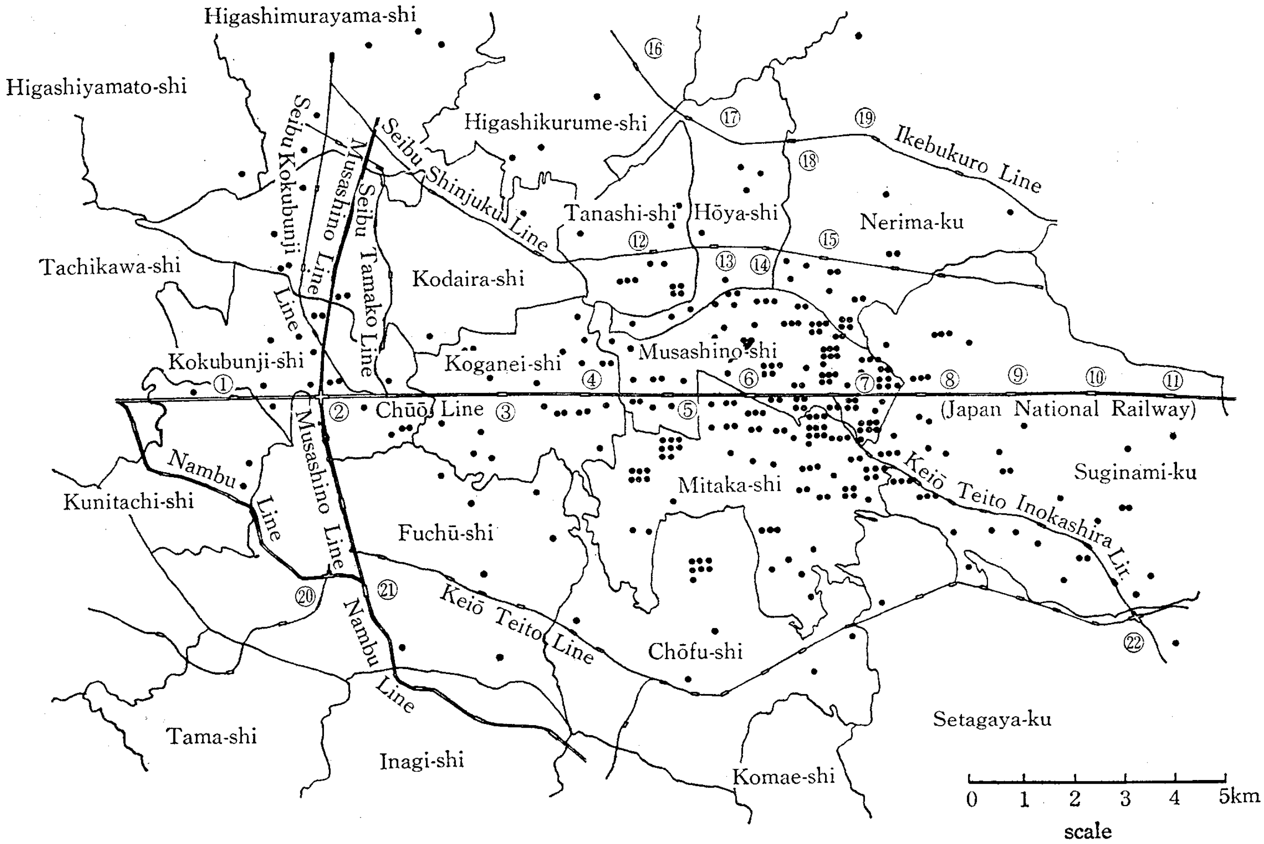


Figure 6 The geographical distribution of customers who came to Kichijōji.

Source : The Data for the Circle of the Kichijōji Station Center.

Notes : This result was obtained by a survey in a single day (Saturday).

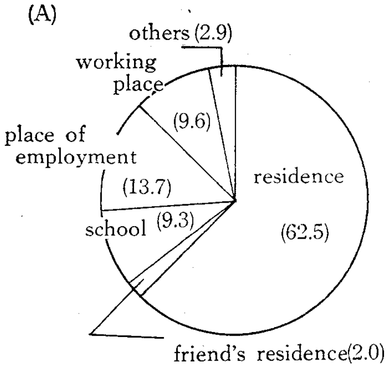
The dots in this map are places of residence of customers who came to Kichijōji. One dot shows the place of residence of one customer. The number of dots in 331. The locations of these dots are obtained from the observation of 409 persons.

The number in a small circle is the number for identifying the main stations in this map. The names of the station given the number are as follows :

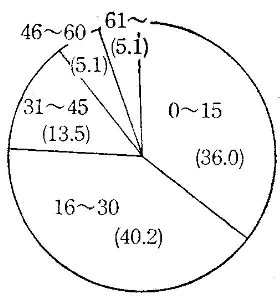
1 : Kunitachi, 2 : Kokubunji, 3 : Musashikoganei, 4 : Higashikoganei, 5 : Musashisakai, 6 : Mitaka, 7 : Kichijōji, 8 : Nishiogikubo, 9 : Ogikubo, 10 : Asagaya, 11 : Kōenji, 12 : Tanashi, 13 : Seibuyagisawa, 14 : Higashifushimi, 15 : Musashisakai, 16 : Higashikurume, 17 : Hoya, 18 : Ōizumigakuen, 19 : Shakujiikōen, 20 : Bubaigawara, 21 : Fuchūhommachi, 22 : Meidaimae.

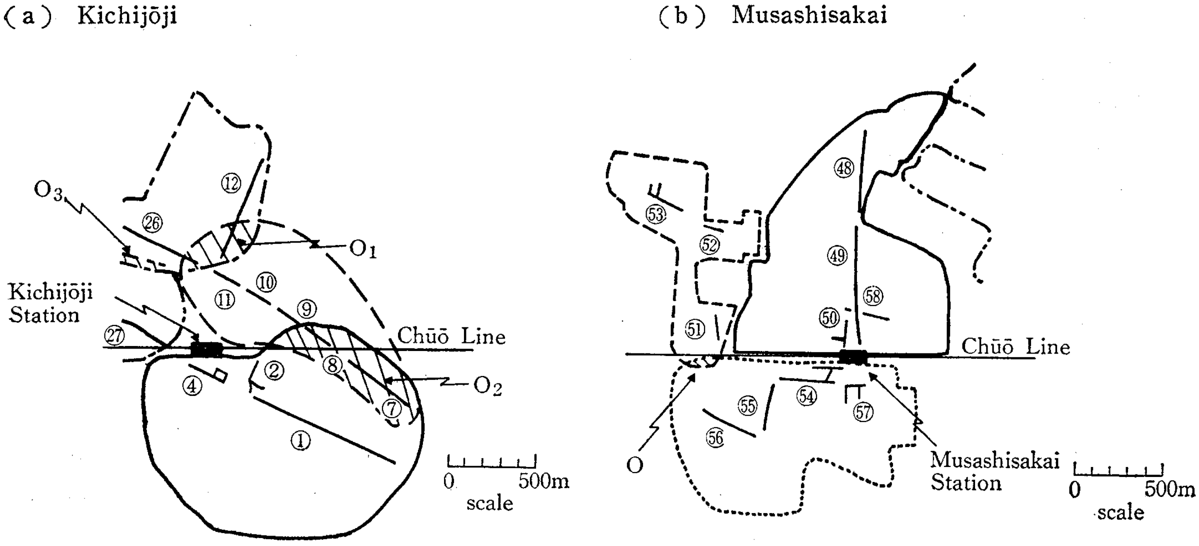
The percentage of the number of customers by places through which they came to Kichijōji and the percentage of the number of customers by time (in minutes) spent for coming to Kichijōji are shown in Figure (A) and (B). The result was obtained from the observation of 408 person. The number in parentheses is the value of the percentage.

(A)



(B)





Boundary lines of market area for shop-lined streets $S(1)$, $S(2)$ and $S(4)$: ——, for shop-lined streets $S(7)$, $S(8)$, $S(9)$, $S(10)$ and $S(11)$: - - -, for shop-lined streets $S(12)$ and $S(26)$: — — — -, and for shop-lined streets $S(27)$ and $S(31)$: — — — —. Overlapping market area : The parts given oblique lines, O_1, O_2 and O_3.

Boundary lines of market area for shop-lined streets $S(48)$, $S(49)$, $S(50)$ and $S(58)$: ——, for shop-lined streets $S(51)$, $S(52)$ and $S(53)$: - - -, for shop-lined streets $S(54)$, $S(55)$, $S(56)$ and $S(57)$: ······, and for other shop-lined streets : — — — — —, — — — — - -. Overlapping market area : A part given oblique lines, O.

Figure 7 The market area for shop-lined streets.

Source : Musashino Shōkō Kaigisho (Musashino Chamber of Commerce and Industry) : Musashinoshi Chiiki Kouri Shōgyō Kindaika Taisaku Chōsa Hōkokusho (Report on the Policy for Modernization of Local Retailers in Musashino City), 1981, p. 50.

Notes : Thick line given a number written in a circle is a shop-lined street. The number written in a circle is the number for identifying shop-lined market. If the number is i, then it shows shop-lined market $S(i)$. The shop-lined street $S(31)$ is not found in the map (a). It is found outside the territory of this map.

overlapping areas appear in the maps is physical insufficiency of the number of the kinds of commodities in a market area, the whole market area will become an overlapping area. Therefore, the emergence of the overlapping areas in these maps may be, at least, partly explained by personal difference of acquisition of information about the spatial distribution of shops——some persons will have an information that they can purchase some kinds of commodities simultaneously in some shops at a spot in market[8], but other persons may not have such an information——and by the mechanism of the emergence of overlapping area depicted by means of the spatial cost line for a commodity among m kinds of commodities, and partly explained by particular personal reason and by the mechanism to be depicted by other principles which may be, for example, sociological or psychological principles of action[9].

6. Conclusion

The traditional theories of market area showed a very lucid explanation of the existence of boundary of market areas for producers of commodities or retailers. These explanation gives us a very important knowledge about the spatial structure of market. However, the theories did not show the existence of overlapping market area[10].

According to the traditional theories, it is difficult to find overlapping market area, provided that they are not given some modification.

I found that if we modify the premises used in the theories, especially the premise about the freight rate, we can theoretically find possibility of existence of overlapping area. The key point of finding overlapping market area was the assumption that when a consumer purchase more than one kind of commodities, the cost of transportation was allocated to the cost of purchasing commodities.

Fortunately, I could show the actual survey of market area in which I can clearly find the overlapping area of retailers, and it was found that the existence of the overlapping area observed here would be explained by the spatial cost line for a commodity among m kinds of commodities proposed here.

7. Acknowledgements

I would like to express my thanks to Professor Masashi Kaneda of Chuo University for his valuable comments and encouragement to this study, Mr. Shigeo Uwabo, Head of Consultation Office of Small and Medium Enterprises, the Musashino Chamber of Commerce and Industry for his explanation of the structure of Kichijōji Shopping Center, and the Musashino Chamber of Commerce and Industry and Kichijōji Station Center for their kindness to allow to use the valuable data obtained from the survey of the behavior of the customers of the Kichijōji Shopping Center and the shop-lined streets in Kichijōji.

要　　約

鈴木啓祐：「多重市場地域の存在に関する理論的考察」『流通経済大学論集』第16巻第3号，1982年，29-38ページ。

市場の内部に複数の生産者あるいは商品販売者が存在するとき，その市場の内部に各生産者または商品販売者の市場地域が形成されるが，その市場形成機構については，この方面の先駆的研究者であるラウンハルト，ホテリング，ならびにパランダーによって，きわめて明確な解明がなされた。これらの先駆者は，いずれも市場地域の存在が運賃によって説明されることを明らかにした。

しかしながら，これらの理論は，同一商品を販売する生産者や販売者の市場が，重複して同一地域に共存するという事実，すなわち，多重市場地域が存在するという事実は説明されていない。

ここでは，伝統的理論に部分的な修正を加え，この多重市場地域の存在の理論的説明を試みた。この地域の存在の理由は，特に，「多数個の商品購入の際の空間費用曲線（spatial cost line for each of n pieces of a commodity)」を基礎として得られる「多数種の商品購入の際の空間費用曲線（spatial cost line for a commodity among m kinds of commodities)」という概念を伝統的理論の中に新たに導入することにより説明される。

幸いにも，国鉄吉祥寺駅および武蔵境駅付近の商店街における最近の商品購入者の行動に関する資料が得られ，この資料から多重市場地域の存在を実際に見いだすことができた。

この研究に有益な助言を加えられた，中央大学の金田昌司教授，この研究に必要な資料を提供された武蔵野商工会議所および株式会社吉祥寺ステーション・センター，ならびに吉祥寺駅周辺の商店街活動に関する

貴重な知識を与えられた武蔵野商工会議所，中小企業　相談所長上保繁雄氏に対し深く感謝の意を表する。

Notes

1) Launhardt, Wilhelm : *Mathematische Regründung der Volkwirtschaftslehre,* Leipzig, 1885.

Isard Walter : *Location and Space-economy,* Massachusetts. The Technology Press of Massachusetts Institute of Technology, 1956, p. 24, pp. 143–171.

Nishioka, Hisao : *Keizai Chiri Bunseki* (*Economic Geographical Analysis*), Tokyo, Taimeido, 1976, p. 241.

Nishioka, Hisao : *Ricchi to Chiiki Keizai* (*Location and Regional Economics*), Tokyo, Miyai Shoten, 1963, p, 354.

Kaneda, Masashi : *Keizai Ricchi to Tochi Riyo* (*Location and Land Use*), Tokyo, Shinhyoron, 1971, pp. 21–27.

Ezawa, Joji : *Ricchi Ron Josetsu* (*Introduction to Location Theory*), Tokyo, Jichō Sha, 1955, pp. 29–32, 179–180.

2) According to the study by Launhardt, the boundary of market area is determined by the following mechanism (Ezawa, Joji *op. cit.*, pp. 179–180).

If the utility obtained from a unit of commodity is denoted by a, the marginal utility of money is w, the price of the commodity at location of the producer of the commodity is p, and the cost of transportion per kilometer (or mile) is f, then the price of the commodity at the location which is z km (kilometer) away from the location of the producer (or supplier) P is given by

$$P=p+fz \qquad (1)$$

Therefore, we can find following relations :

$$\frac{a}{p+fz}=w \qquad (2)$$

$$z=\frac{1}{f}\left(\frac{a}{w}-p\right) \qquad (3)$$

On the other hand, if the highest price of the commodity which is accepted by consumer is $\bar{P}$, the $\bar{P}$ must be expressed by the following equation :

$$\bar{P}=\frac{a}{w} \qquad (4)$$

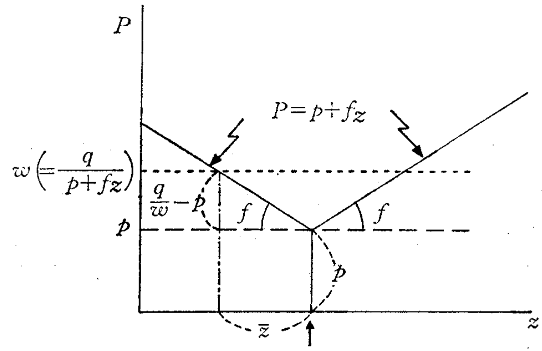
Using these premises, Launhardt, concluded that the distance between the location of producer (or supplier) R and the boundary of the market area of the commodity, $\bar{z}$ is given by

$$\bar{z}=\frac{\bar{P}-p}{f} \qquad (5)$$

and the area of the market area A is given by

$$A=\pi\bar{z}^2$$
$$=\pi\left(\frac{\bar{P}-p}{f}\right)^2 \qquad (6)$$

In the figure shown below, we can find the values given in the premises of this discussion



the location of producer
(or supplier) of a commodity

and $\bar{z}$.

3) Hotelling, Harold : "Stubility in Competition," *Economic Journal*, Vol. 39, 1929, pp. 41–57.
Vickrey, William S. : *Microstatics*, New York, Harcourt, Brace & World, 1964, pp. 323–334.
Smith, David M. : *Industrial Location*, New York, John Wiley & Sons, 1971, pp. 138–143.

4) Palander, Tord : *Beitrage zur Standortstheorie*, Uppsala, Almqvist & Wiksells Boktryckeri, 1935.
Smith, David M. : *op. cit.*, pp. 119–124.

5) According to the theories presented by Palander and Hotelling, we cannot find overlapping market area, because the market area observed is devided by a point or points.
In Launhardt's discussion, overlapping market area for two producers cannot be found, because in his discussion, a market area for only one producer is treated, as shown in note 2).

6) The result of this survey is found in the *Data for the Circles of the Kichijōji Station Center.* In the area where the consumers, who came to Kichijōji were found, we can find other towns which have shop-lined streets like those found in Kichijōji. For example, Asagaya, Ogikubo, Mitaka, Kokubunji, and Tanashi are the towns, where we can easily buy most of the commodities which are found in Kichijōji.

7) Musashino-shi Chiiki Kouri Shōgyō Kindaika Taisaku Chōsa Hōkokusho (Report on the Policy for Modernization of Local Retailers in Musashino City), 1981, p. 50.

8) A person who has an information that they can purchase some kinds of commodities simultaneously in some shops at a spot in a market, he may go to the spot to purchase the commodities and his action may become a reason for the existence of overlapping market area.

9) Some person may know the existence of a favorite shop managed by his friend, or some person may know the existence of a favorite shop in which commodities for sale are neatly arranged. And if they go to purchase commodities to these favorite shops, even though the shops are considerably far from their residences, their actions may become a reason for the existence of overlapping area.

10) The traditional theories showed only a very clear boundaries of market area.